Optimal Multiprocessor Locking Protocols under FIFO Scheduling

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Abstract

Real-time locking protocols are typically designed to reduce any *priority-inversion* blocking (pi-blocking) a task may incur while waiting to access a shared resource. For the multiprocessor case, a number of such protocols have been developed that ensure *asymptotically* optimal pi-blocking bounds under job-level fixed-priority scheduling. Unfortunately, no optimal multiprocessor real-time locking protocols are known that ensure *tight* pi-blocking bounds under any scheduler. This paper presents the first such protocols. Specifically, protocols are presented for mutual exclusion, reader-writer synchronization, and k-exclusion that are optimal under first-in-first-out (FIFO) scheduling when schedulability analysis treats suspension times as computation. Experiments are presented that demonstrate the effectiveness of these protocols.

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1 Introduction

In recent years, a number of suspension-based multiprocessor real-time locking protocols have been developed that provide asymptotically optimal bounds on priority-inversion blocking (pi-blocking) under suspension-oblivious (s-oblivious) schedulability analysis, which treats suspension time analytically as computation time [11, 13, 14]. For mutual-exclusion (mutex) sharing, most (if not all) known asymptotically optimal locking protocols ensure a per-task s-oblivious pi-blocking bound of $2m-1$ request lengths on an m-processor platform under job-level fixed-priority (JLFP) scheduling [11, 13].[1] The commonality of this bound is somewhat surprising as these protocols include ones that target different scheduling strategies (*e.g.*, partitioned, global, and clustered scheduling) and employ different mechanisms to cope with pi-blocking (*e.g.*, priority inheritance vs. priority donation [11, 13]).

In contrast, under s-oblivious analysis, the current best lower bound yields a worst-case per-task pi-blocking bound of at least $m-1$ request lengths [11]. This gap between the existing lower bound and upper bound raises an obvious question: *is a pi-blocking bound of* $2m-1$ *request lengths fundamental under JLFP scheduling?*

In this paper, we answer this question negatively by showing that, under s-oblivious analysis, the existing lower bound of $m-1$ request lengths is tight under *first-in-first-out* (FIFO) scheduling. To show this, we give a suspension-based locking protocol for mutex sharing that ensures a per-lock-request s-oblivious pi-blocking bound of at most $m-1$ request lengths under FIFO scheduling, matching the lower bound. Our protocol is designed for

[1] We refine this statement later by distinguishing between *request blocking* and *release blocking*.

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clustered scheduling, so it can be applied under global and partitioned scheduling as well. To our knowledge, this is the first *truly optimal* suspension-based multiprocessor locking protocol under any practical scheduling algorithm.

In designing our protocol, we exploit the fact that independent (non-resource-sharing) tasks are non-preemptive under FIFO scheduling. Such non-preemptivity is a property of the scheduler itself and does not have to be otherwise enforced: under FIFO scheduling, a newly released instance of a task cannot cause any other task instance to have insufficient priority to be scheduled. Asymptotically optimal locking protocols such as the C-OMLP [13] enforce such a property via an explicit progress mechanism. We show that such mechanisms are not required under FIFO scheduling.

Our locking protocol strengthens the case for using FIFO scheduling on multiprocessors. In addition to enabling a tight pi-blocking bound, FIFO scheduling has low overheads, ensures bounded response times (and hence bounded deadline tardiness in soft real-time systems) without capacity loss [2, 22], and is *sustainable* with respect to execution times, meaning that it is safe to perform schedulability analysis assuming all instances of a task take its *worst-case execution time* (WCET) to complete. Moreover, non-preemptive execution also eases the determination of WCETs, which is challenging on modern multiprocessors [31]. According to a recent survey, around 30% of industrial respondents reported using FIFO scheduling [3].

Contributions. Our contributions are fourfold.

First, we propose a suspension-based mutex locking protocol called the *optimal locking protocol under FIFO scheduling* (OLP-F). The OLP-F restricts a task from issuing a resource request until it has high enough priority. Together with properties of FIFO scheduling, this ensures that the OLP-F has a tight s-oblivious pi-blocking bound under FIFO scheduling.

Second, we consider an extension of mutex sharing called *k-exclusion* sharing, which permits k simultaneous lock holders. For k-exclusion, we propose the *optimal locking protocol for k-exclusion under FIFO scheduling* (k-OLP-F) and show that it has a tight s-oblivious pi-blocking bound under FIFO scheduling.

Third, we expand even further beyond mutex sharing by considering *reader-writer* (RW) sharing, where exclusive resource usage is only required for write accesses and concurrent read accesses are permitted. For RW sharing, we propose the *read-optimal RW locking protocol under FIFO scheduling* (RW-OLP-F), which provides a tight s-oblivious pi-blocking bound for read requests under FIFO scheduling. Additionally, under the RW-OLP-F, the pi-blocking bound for write requests is just under two request lengths of optimal.

Finally, we provide experimental results that show the benefits of our locking protocols.

Organization. In the rest of this paper, we provide needed background (Sec. 2), delve further into s-oblivious pi-blocking (Sec. 3), establish a FIFO-based progress property for resource sharing (Sec. 4), present the above-mentioned protocols (Secs. 5–7), present our experimental results (Sec. 8), more fully review related work (Sec. 9), and conclude (Sec. 10).

2 System Model and Background

In this section, we provide needed definitions; Tbl. 1 summarizes the notation given here.

Task model. We consider a system of n sporadic *tasks* $\tau_1, \tau_2, \ldots, \tau_n$ to be scheduled on m identical processors by a FIFO scheduler. Each task τ_i releases a potentially infinite sequence of *jobs* $J_{i,1}, J_{i,2}, \ldots$. (We omit job indices if they are irrelevant.) Each task τ_i has a *period* T_i specifying the minimum spacing between consecutive job releases. Each task has a *relative*

Table 1 Notation summary.

Symbol	Meaning	Symbol	Meaning
n	Number of tasks	ℓ_q	q^{th} shared resource
m	Number of processors	N_i^q	Maximum number of requests for ℓ_q by τ_i
τ_i	i^{th} task	L_i^q	Maximum request length for ℓ_q by τ_i
$J_{i,j}$	j^{th} job of τ_i	L^q_{max}	$\max_{1\le i\le n}\{L_i^q\}$
T_i	Period of τ_i	L_{max}	$\max_{1\le q\le n_r} L^q_{max}$
C_i	WCET of τ_i	$\mathcal{R}$	A request
D_i	Relative deadline of τ_i	$r_{i,j}$	Release time of $J_{i,j}$
u_i	Utilization of τ_i	$f_{i,j}$	Finish time of $J_{i,j}$
n_r	Number of resources	$L^q_{sum,h}$	sum of the h highest L_i^q values

deadline D_i. Task τ_i has an *implicit deadline* if $D_i = T_i$, a *constrained deadline* if $D_i \le T_i$, and an *arbitrary deadline* if no relationship between D_i and T_i is assumed. Each task has a WCET denoted C_i. Task τ_i's *utilization* is defined as $u_i = C_i/T_i$.

The *release time* (resp., *finish time*) of a job $J_{i,j}$ is given by $r_{i,j}$ (resp., $f_{i,j}$). $J_{i,j}$ is *pending* at time t if $r_{i,j} \le t < f_{i,j}$. Jobs of a task τ_i are sequential, *i.e.*, $J_{i,j+1}$ cannot commence execution before $J_{i,j}$ finishes. Job $J_{i,j}$ is *eligible* to execute at time t if $J_{i,j}$ is pending at time t and $t \ge f_{i,j-1}$ holds (if $j > 1$). An eligible job is either *ready* (when it can be scheduled) or *suspended* (when it cannot be scheduled).

We assume time to be discrete and a unit of time to be 1.0. All scheduling decisions are taken at integer points in time. We also assume all task parameters to be integers.

Multiprocessor scheduling. Multiprocessor scheduling approaches can be broadly classified into two categories: *partitioned* and *global*. Under partitioned scheduling, a task is statically assigned to a processor and cannot migrate to another processor. Global scheduling allows a task to execute on any of the m processors. *Clustered scheduling* is a hybrid of partitioned and global scheduling. Under clustered scheduling, all m processors are partitioned into $m/c \in \mathbb{N}$ clusters (without loss of generality, we assume m is an integer multiple of c) each containing c processors.[2] Each task is assigned to a cluster and can migrate only among the processors of the cluster. We consider clustered scheduling, as both partitioned and global scheduling are special cases ($c = 1$ and $c = m$, respectively).

Under a *job-level fixed-priority* (JLFP) scheduler, each job is assigned a fixed priority throughout its execution, but a task's priority may change over time. Common JLFP schedulers include *earliest-deadline-first* (EDF), FIFO, and *fixed-priority* scheduling algorithms. When such algorithms are employed with clustered scheduling, the c highest-priority ready jobs (if that many exist) of each cluster are scheduled on the processors of that cluster. In this paper, we consider clustered FIFO (C-FIFO) scheduling where, within a cluster, jobs with earlier release times have higher priority. We assume ties are broken arbitrarily but consistently. Hereafter, we assume all schedules to be C-FIFO unless otherwise stated.

Resource model. We consider a system that has a set $\{\ell_1, \ldots, \ell_{n_r}\}$ of shared resources. For now, we limit attention to *mutual exclusion* (mutex) sharing, although other notions of sharing will be considered later. Under mutex sharing, a resource ℓ_q can be held by at most one job at any time. When a job J_i requires a resource ℓ_q, it *issues* a *request* $\mathcal{R}$ for ℓ_q. $\mathcal{R}$ is *satisfied* as soon as J_i *holds* ℓ_q, and *completes* when J_i *releases* ℓ_q. $\mathcal{R}$ is *active* from its issuance to its completion. J_i must *wait* until $\mathcal{R}$ can be satisfied if it is held by another job.

[2] Our results can be adapted for non-uniform cluster sizes at the expense of additional notation.

Table 2 Asymptotically optimal locking protocols for mutex locks under s-oblivious analysis.

Scheduling	Protocol	Release blocking	Request blocking
Global JLFP	OMLP [11]	0	$(2m-1)L^q_{max}$
Clustered JLFP	C-OMLP [13]	mL_{max}	$(m-1)L^q_{max}$
Clustered JLFP	OMIP [7]	0	$(2m-1)L^q_{max}$
C-FIFO	OLP-F (This work)	0	$(m-1)L^q_{max}$

It may do so either by *busy-waiting* (or *spinning*) in a tight loop, or by being *suspended* by the operating system (OS) until $\mathcal{R}$ is satisfied. We assume that if a job J_i holds a resource ℓ_q, then it must be scheduled to execute.[3] A resource access is called a *critical section* (CS).

We assume that each job can request or hold at most one resource at a time, *i.e.*, resource requests are non-nested. We let N_i^q denote the maximum number of times a job of task τ_i requests ℓ_q, and let L_i^q denote the maximum length of such a request. We define L_i^q to be 0 if $N_i^q = 0$. Finally, we define $L^q_{max} = \max_{1\leq i\leq n}\{L_i^q\}$, and $L_{max} = \max_{1\leq q\leq n_r}\{L^q_{max}\}$, and let $L^q_{sum,h}$ be the sum of the h largest L_i^q values. We assume all L_i^q and N_i^q to be constant.

Priority inversions. *Priority-inversion blocking* (or *pi-blocking*) occurs when a job is delayed and this delay cannot be attributed to higher-priority demand for processing time. Under a given real-time locking protocol, a job may experience pi-blocking each time it requests a resource—this is called *request blocking*—and/or upon its release and each time it releases a resource—this is called *release blocking*.

On multiprocessors, the formal definition of pi-blocking actually depends on how schedulability analysis is done. Of relevance to suspension-based locks, schedulability analysis may be either *suspension-oblivious* (*s-oblivious*) or *suspension-aware* (*s-aware*) [11]. Under s-oblivious analysis (the focus of this work), suspension time is *analytically* treated as computation time.

Blocking complexity. Request lengths are unavoidable in assessing maximum pi-blocking, as a request-issuing job may have to wait for a current resource-holder to complete before its request can be satisfied. As such, maximum pi-blocking bounds are usually expressed as an integer multiple of the maximum request length, *i.e.*, the number of requests that are satisfied while a resource-requesting job is pi-blocked.

Asymptotically optimal locking protocols. For mutex locks, Brandenburg and Anderson established a lower bound of $m-1$ request lengths on per-request s-oblivious pi-blocking under any JLFP scheduler [11]. Thus, under s-oblivious analysis, an asymptotically optimal locking protocol achieves $O(m)$ per-job pi-blocking. Locking protocols such as the OMLP [11], the OMIP [7], and the C-OMLP [13] are asymptotically optimal under JLFP scheduling. Tbl. 2 provides a summary of existing asymptotically optimal locking protocols.[4]

Optimal locking protocols. We call a locking protocol *optimal* under a scheduling algorithm if it ensures a pi-blocking bound that is tight, *i.e.*, it matches the lower bound on pi-blocking under that scheduling algorithm.

[3] This is a common assumption in work on synchronization. It is needed for shared data, but may be pessimistic for other shared resources such as I/O devices.
[4] Note that, for the C-OMLP, the $2m-1$ bound mentioned in Sec. 1 comes from a combination of release and request blocking.

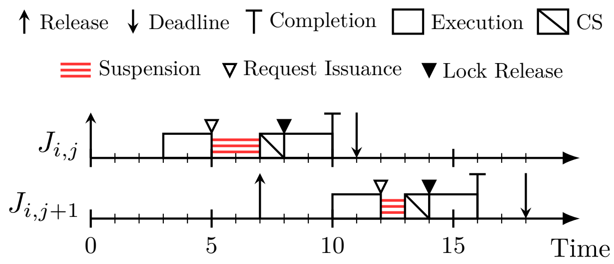


Figure 1 A schedule illustrating s-oblivious pi-blocking for arbitrary-deadline tasks.

3 Suspension-Oblivious Pi-Blocking

Under s-oblivious schedulability analysis, each task's WCET is inflated by the amount of worst-case s-oblivious pi-blocking any of its jobs may suffer. Such s-oblivious pi-blocking was originally defined for implicit-deadline hard real-time systems [11]. In this section, we show that this definition needs refinement for systems with arbitrary deadlines or soft timing constraints. We also provide a refined definition that works under such cases. We begin by reviewing the original definition of s-oblivious pi-blocking under clustered scheduling.

▶ **Definition 1** ([11]). *Under s-oblivious schedulability analysis, a job J_i incurs s-oblivious pi-blocking at time t if J_i is* ***pending*** *but not scheduled and fewer than c higher-priority jobs are* ***pending*** *in its cluster.*

If tasks have arbitrary deadlines or can miss their deadlines due to soft timing constraints, Def. 1 may inappropriately identify certain delays due to the sequential execution of tasks as s-oblivious pi-blocking. The following example illustrates this.

▶ **Example 2.** Fig. 1 illustrates two consecutive jobs $J_{i,j}$, and $J_{i,j+1}$ of a task τ_i with $T_i = 7$ and $D_i = 11$. Job $J_{i,j+1}$ is released at time 7 and job $J_{i,j}$ finishes execution at time 10. Thus, job $J_{i,j+1}$ is pending but not eligible during the time interval $[7, 10)$. Assume that both $J_{i,j}$ and $J_{i,j+1}$ are among the c highest-priority pending jobs in their cluster during $[7, 10)$. Assuming $c > 1$, by Def. 1, $J_{i,j+1}$ is s-oblivious pi-blocked during the interval $[7, 10)$. However, $J_{i,j+1}$'s delay during $[7, 10)$ is not due to a locking-related suspension. Under s-oblivious schedulability analysis, it is not necessary to inflate τ_i's WCET to include such a delay. In fact, doing so may cause a circular problem, *i.e.*, the inflated WCET may cause additional delays, which can then necessitate further inflation.

The above example motivates refining the notion of s-oblivious pi-blocking as follows.

▶ **Definition 3.** *Under s-oblivious schedulability analysis, a job J_i incurs s-oblivious pi-blocking at time t if J_i is* ***eligible*** *but not scheduled and fewer than c higher-priority jobs are* ***eligible*** *in its cluster.*

▶ **Example 2 (Cont'd).** $J_{i,j+1}$ is pending but not eligible during the interval $[7, 10)$. Thus, it is not s-oblivious pi-blocked during that interval. However, $J_{i,j+1}$ is eligible during $[12, 13)$. Assume that $J_{i,j+1}$ is among the c highest-priority eligible jobs during $[12, 13)$, but is suspended. Then, by Def. 3, $J_{i,j+1}$ is s-oblivious pi-blocked during $[12, 13)$.

4 Resource-Holder's Progress Under FIFO Scheduling

Any real-time locking protocol needs to ensure a resource-holding job's progress whenever a job waiting for the same resource is pi-blocked, for otherwise, the maximum per-job pi-blocking can be very large or even unbounded. To ensure that the maximum pi-blocking is reasonably bounded, real-time locking protocols employ *progress mechanisms* that may temporarily raise a job's *effective priority.* One such mechanism is *priority inheritance* [26,28], which raises the effective priority of a job holding resource ℓ_q to the maximum of its priority and the priorities of all jobs waiting for ℓ_q. Another alternative is *priority donation* [14], which ensures that a job J_i can only issue a request when its priority is high enough to be scheduled. Moreover, if a job J_j's release causes J_i to have insufficient priority to be scheduled, then J_j "donates" its priority to J_i. This ensures that a resource holder is always scheduled. This property makes priority donation particularly effective under clustered scheduling.

Progress under FIFO scheduling. The above-mentioned progress mechanisms can be utilized to design multiprocessor locking protocols that are asymptotically optimal under any JLFP scheduling policy [11, 14]. Interestingly, for the case of C-FIFO scheduling, no such progress mechanism is required to design optimal locking protocols. In fact, the C-FIFO scheduling policy itself has properties that ensure the progress of a resource-holding job. The key property that enables such progress is given in the following lemma.

▶ **Lemma 4.** *Under C-FIFO scheduling, if a job $J_{i,j}$ becomes one of the c highest-priority eligible jobs in its cluster at time t_h, then it remains so during $[t_h, f_{i,j})$.*

Proof. Assume for a contradiction that t is the first time instant in $[t_h, f_{i,j})$ such that $J_{i,j}$ is not one of the c highest-priority eligible jobs in its cluster. Then, $t > t_h$ holds. By the definition of time t, there are at most $c-1$ (resp., at least c) eligible jobs with higher priority than $J_{i,j}$ at time $t-1 \geq t_h$ (resp., t) in $J_{i,j}$'s cluster. Thus, there is a task τ_u that has an eligible job $J_{u,v}$ with higher priority than $J_{i,j}$ at time t, but has no such job at time $t-1$.

Since $J_{u,v}$'s priority exceeds $J_{i,j}$'s, $r_{u,v} \leq r_{i,j}$ holds. Since $J_{i,j}$ is eligible at time t_h, $r_{i,j} \leq t_h$ holds. Thus, $r_{u,v} \leq t_h$ and $J_{u,v}$ is pending at time $t-1$. We now consider two cases.

Case 1. $v = 1$. In this case, $J_{u,v}$ is also eligible at time $t-h$. Thus, τ_u has an eligible job with higher priority than $J_{i,j}$ at time $t-1$, a contradiction.

Case 2. $v > 1$. Since $J_{u,v}$ is not eligible at time $t-1$, job $J_{u,v-1}$ is eligible at time $t-1$. We have $r_{u,v-1} < r_{u,v} \leq r_{i,j}$. Thus, τ_u has an eligible job with higher priority than $J_{i,j}$ at time $t-1$, a contradiction.

Therefore, we reach a contradiction in both cases. ◀

Utilizing Lemma 4, we have the following lemma.

▶ **Lemma 5.** *If a job $J_{i,j}$ issues a request $\mathcal{R}$ when it is one of the c highest-priority jobs in its cluster, then $J_{i,j}$ is always scheduled from $\mathcal{R}$'s satisfaction to completion.*

Proof. Let t_r, t_s, and t_c be the time instants when $\mathcal{R}$ is issued, satisfied and complete, respectively. Thus, $t_r \leq t_s \leq t_c$ holds. Since $J_{i,j}$ is one of the c highest-priority eligible jobs in its cluster at time t_r, by Lemma 4, $J_{i,j}$ remains one of the c highest-priority eligible jobs in its cluster throughout $[t_r, t_c)$. Since $\mathcal{R}$ is satisfied at time $t_s \geq t_r$, $J_{i,j}$ is ready throughout $[t_s, t_c)$. Thus, $J_{i,j}$ is scheduled during $[t_s, t_c)$. ◀

Thus, by requiring a request to be issued only when the request-issuing job is one of the top-c-priority jobs in its cluster, we can ensure a resource-holder's progress under FIFO scheduling. We exploit this property in designing our protocols. Note that the C-OMLP ensures this property by employing priority donation as its progress mechanism at the expense of additional release blocking that may be incurred by a job even if it does not require any resource [13]. Due to this, our protocols have features in common with the C-OMLP.

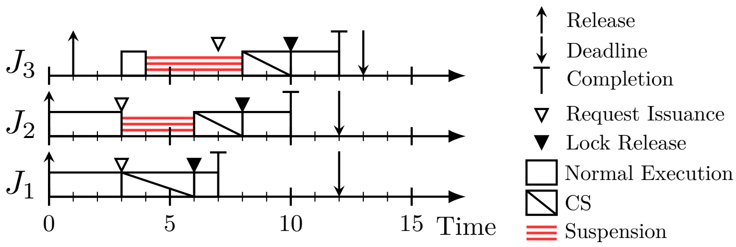


Figure 2 A schedule illustrating the OLP-F.

5 Mutex Locks

In this section, we introduce the *optimal locking protocol for mutual exclusion sharing under C-FIFO scheduling* (OLP-F), which achieves optimal pi-blocking under C-FIFO scheduling. To match the lower bound on pi-blocking, the OLP-F ensures that each job suffers pi-blocking for the duration of at most $m-1$ request lengths and incurs no release blocking.

Structures. For each resource ℓ_q, we have a FIFO queue FQ_q that contains requests for ℓ_q. A request $\mathcal{R}$ is satisfied if and only if $\mathcal{R}$ is the head of the FQ_q.

Rules. When a job J_i attempts to issue a request $\mathcal{R}$ for a resource ℓ_q, it proceeds according to the following rules.

M1 J_i is allowed to issue $\mathcal{R}$ only if it is one of the c highest-priority eligible jobs in its cluster. J_i suspends if necessary to ensure this condition.

M2 When J_i issues $\mathcal{R}$, $\mathcal{R}$ is enqueued in FQ_q. If J_i becomes the head of FQ_q, then it is immediately satisfied. Otherwise, it suspends.

M3 $\mathcal{R}$ is satisfied when it is the head of FQ_q. $\mathcal{R}$ is removed from the FQ_q when it is complete.

▶ **Example 6.** Fig. 2 illustrates a C-FIFO schedule of three jobs on a two-processor cluster. J_1 and J_2 are released earlier (hence, have higher priorities) than J_3. Both J_1 and J_2 issue requests for resource ℓ_q at time 3 and J_1's request is enqueued first. Assuming no job in a different cluster holds ℓ_q, J_1 acquires ℓ_q at time 3 by Rule M2. At time 3, since J_2 is suspended, J_3 starts to execute. At time 4, J_3 attempts to issue a request for ℓ_q, but it is suspended due to Rule M1 as it is not one of the top-2-priority jobs at that time. At time 6, J_1 releases ℓ_q and J_2's request is satisfied according to Rule M3. Since J_3 becomes one of the top-2-priority jobs when J_1 completes, it issues a request for ℓ_q at time 7.

Analysis. To derive an upper bound on the pi-blocking suffered by a job, we first show that FQ_q contains no more than m requests at any time.

▶ **Lemma 7.** *Under the OLP-F, at any time, FQ_q contains at most m requests.*

Proof. Assume that t is the first time instant when FQ_q contains more than m requests. Each job has at most one active request at any time. Thus, at time t, FQ_q must contain a request $\mathcal{R}$ issued by a job J_i that is not one of the c highest-priority eligible jobs in its cluster. Let $t' \le t$ be the time instant when J_i issues $\mathcal{R}$. By Rule M1, J_i is one of the c highest-priority eligible jobs in its cluster at time t'. Since J_i is not complete at time t, by Lemma 4, it is one of the c highest-priority eligible jobs in its cluster at time t, a contradiction. ◀

We now determine an upper bound on the request blocking suffered by job J_i when it issues a request $\mathcal{R}$ for resource ℓ_q. Fig. 3 depicts the timeline of $\mathcal{R}$ from when J_i attempts

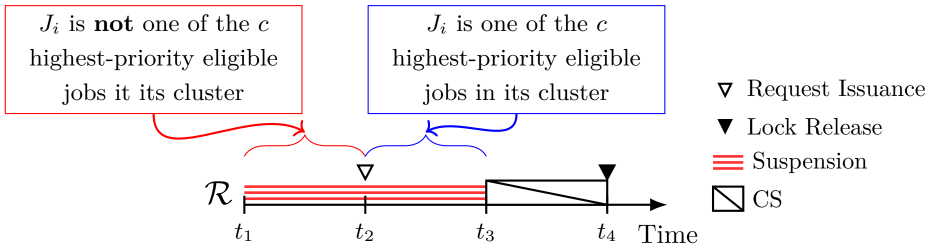


Figure 3 Timeline of a request under the OLP-F.

to issue $\mathcal{R}$ to when $\mathcal{R}$ completes. Let t_1 be the time instant when job J_i attempts to issue request $\mathcal{R}$. Let t_2 be the first time instant at or after time t_1 when J_i, becomes one of the top-m-priority eligible jobs. Therefore, by Rule M1, $\mathcal{R}$ is issued at time t_2. Let t_3 and t_4 be the time instants when $\mathcal{R}$ is satisfied and completes, respectively.

▶ **Lemma 8.** *During $[t_1, t_3]$, J_i incurs pi-blocking for at most $L^q_{sum,m-1}$ time units.*

Proof. By the definition of t_2, J_i is not one of the top-c-priority eligible jobs in its cluster during $[t_1, t_2)$. Hence, J_i is not pi-blocked during that time. By Lemma 4, J_i is pi-blocked throughout $[t_2, t_3)$. By Lemma 5, J_i is continuously scheduling during $[t_3, t_4)$. Thus, from t_1 to t_4, J_i is only pi-blocked during $[t_2, t_3)$.

By Lemma 7, at most $m-1$ other requests precede $\mathcal{R}$ in FQ_q at time t_2. By Rule M3 and Lemma 5, each job at the head of FQ_q is continuously scheduled until its request is complete. Since each task has at most one eligible job and each job has at most one request at any time, $t_3 - t_2$ is not more than $L^q_{sum,m-1}$ time units and the lemma follows. ◀

We now show that the OLP-F does not cause any release blocking under C-FIFO scheduling.

▶ **Lemma 9.** *Under the OLP-F, no job incurs release blocking.*

Proof. Since a resource-holding job is scheduled only when its priority is among the top c in its cluster, a resource request $\mathcal{R}$ does not cause pi-blocking to any job (within and across cluster boundaries) that does not issue a request during the time $\mathcal{R}$ is satisfied. ◀

▶ **Theorem 10.** *Under the OLP-F, J_i is pi-blocked for at most $b_i = \sum_{q=1}^{n_r} N^q_i \cdot L^q_{sum,m-1}$ time units.*

Proof. Follows from Lemmas 8 and 9. ◀

Thus, the OLP-F is an optimal locking protocol under C-FIFO scheduling.

6 *k*-Exclusion Locks

k-exclusion generalizes mutual exclusion by allowing up to k simultaneous lock holders; thus, mutual exclusion is equivalent to 1-exclusion. In this section, we give an optimal k-exclusion locking protocol under C-FIFO scheduling. We assume that a resource ℓ_q can be concurrently held by up to $k_q \leq m$ jobs. We begin by reviewing lower bound results for k-exclusion.

Lower bound on pi-blocking. For k-exclusion, Elliot *et al.* showed that a task system and a release sequence for it exist such that a job requesting a resource ℓ_q incurs s-oblivious pi-blocking for the duration of $\lceil \frac{m-k_q}{k_q} \rceil$ request lengths under any JLFP scheduler [18].

Table 3 Asymptotically optimal locking protocols for k-exlcusion locks under s-oblivious analysis.

Scheduling	Protocol	Release blocking	Request blocking
Clustered JLFP	CK-OMLP [11]	$\max_q\{\lceil m/k_q\rceil L^q_{max}\}$	$(\lceil m/k_q\rceil - 1)L^q_{max}$
Global JLFP	OKGLP [18]	0	$(2\lceil m/k_q\rceil + 4)L^q_{max}$
Global JLFP	R^{2}DGLP [30]	0	$(2\lceil m/k_q\rceil - 2)L^q_{max}$
C-FIFO	k-OLP-F (This work)	0	$(\lceil m/k_q\rceil - 1)L^q_{max}$

Asymptotically optimal locking protocols. Under s-oblivious analysis, the CK-OMLP [11], the OKGLP [18], and the R^{2}DGLP [30] ensure asymptotically optimal pi-blocking for k-exclusion. Tbl. 3 summarizes these protocols.

The k-OLP-F. We now introduce the *optimal locking protocol for k-exclusion under C-FIFO scheduling* (k-OLP-F), which achieves optimal pi-blocking for k-exclusion under C-FIFO scheduling. The k-OLP-F ensures that a job suffers pi-blocking for the duration of no more than $\lceil \frac{m-k_q}{k_q} \rceil$ request lengths for each request for ℓ_q and incurs no release blocking.

Structures. For each resource ℓ_q, we have a FIFO queue FQ$_q$ that contains *waiting* requests for ℓ_q. We also have a queue SQ$_q$ of length at most k_q that contains the *satisfied* requests for ℓ_q. Initially, both queues are empty. A request $\mathcal{R}$ is satisfied if and only if $\mathcal{R}$ is in SQ$_q$.

Rules. When a job J_i attempts to issue a request $\mathcal{R}$ for a resource ℓ_q, it proceeds according to the following rules.

K1 J_i is allowed to issue $\mathcal{R}$ only if J_i is one of the c highest-priority eligible jobs in its cluster. J_i suspends if necessary to ensure this condition.

K2 If the length of SQ$_q$ is less than k_q when J_i issues $\mathcal{R}$, then $\mathcal{R}$ is enqueued in SQ$_q$ and is immediately satisfied. Otherwise, $\mathcal{R}$ is enqueued in FQ$_q$ and J_i suspends.

K3 When $\mathcal{R}$ completes, it is removed from SQ$_q$. If FQ$_q$ is non-empty at that time, then the head of FQ$_q$ is dequeued, enqueued in SQ$_q$, and satisfied.

▶ **Example 11.** Fig. 4 shows a schedule of five jobs that share a resource ℓ_q with $k_q = 2$. Jobs J_1, J_2, and J_3 (resp., J_4, and J_5) are FIFO scheduled on a two-processor cluster G_1 (resp., G_2). Since SQ$_q$ is initially empty, by Rule K2, J_4 and J_1 acquire ℓ_q at times 2 and 3, respectively. Since both J_2 and J_5 are one of the top-2-priority eligible jobs in their clusters, by Rule K1, they issue requests for ℓ_q at times 4 and 5, respectively. At time 5, J_3 attempts to issue a request for ℓ_q, but is suspended, by Rule K1. At time 5, J_4 releases ℓ_q and is removed from SQ$_q$ by Rule K3. J_2's request is at the head of FQ$_q$ at time 5, so by Rule K3, it is removed from FQ$_q$, enqueued in SQ$_q$, and satisfied. At time 7, J_1 completes and J_3 becomes one of the top-2-priority jobs in G_1 and issues its request, by Rule K1.

Analysis. We now derive an upper bound on the pi-blocking suffered by a job under the k-OLP-F. We first derive an upper bound on the number of waiting requests in FQ$_q$.

▶ **Lemma 12.** *Under the k-OLP-F, FQ$_q$ contains at most $m - k_q$ requests.*

Proof. Assume otherwise. Let t be the first time instant such that FQ$_q$ contains more than $m - k_q$ requests. Thus, a new request $\mathcal{R}'$ is enqueued in FQ$_q$ at time t. By Rule K2, SQ$_q$ contains k_q requests at time t. Thus, the number of active requests (either satisfied or waiting) is more than $k_q + m - k_q = m$ at time t. Since each job has at most one active request at any time, there is an active request $\mathcal{R}$ issued by a job J_i that is not one of the c highest-priority jobs in its cluster. By Rule K1, J_i is one of the c highest-priority jobs in its cluster when it issues $\mathcal{R}$ at time $t' \leq t$. By Lemma 4, J_i remains as one of the c highest-priority jobs in its cluster at time t, a contradiction. ◀

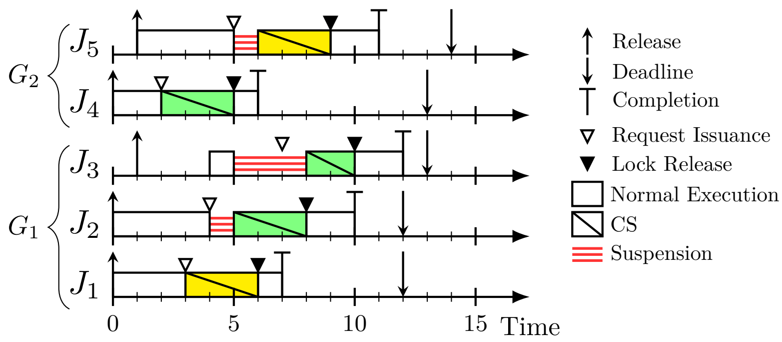


Figure 4 A schedule illustrating the k-OLP-F. Concurrent resource accesses are shaded differently.

We now determine an upper bound on request blocking. We consider a job J_i that issues a request $\mathcal{R}$ for resource ℓ_q. As in Fig. 3, let t_1, t_2, t_3, and t_4 be the time instants corresponding to when J_i attempts to issue $\mathcal{R}$, and when $\mathcal{R}$ is issued, satisfied, and complete, respectively.

▶ **Lemma 13.** *For request $\mathcal{R}$, J_i suffers request blocking for at most $L^q_{sum,\lceil \frac{m-k_q}{k_q} \rceil}$ time units.*

Proof. By Def. 3, J_i does not suffer any pi-blocking during $[t_1, t_2)$ and $[t_3, t_4)$. By Lemma 4 and the definition of t_2, J_i suffers pi-blocking during the entire duration of $[t_2, t_3)$, so it suffices to upper bound $(t_3 - t_2)$. If SQ_q contains fewer than k_q requests at time t_2, then $t_3 - t_2 = 0$ holds by Rule K2, so assume otherwise. At time t_2, no two requests in SQ_q and FQ_q are from the same task. By Rule K3, $\mathcal{R}$ is satisfied when it is dequeued from FQ_q. Thus, by Lemma 12, at most $m - k_q$ requests are required to be dequeued to satisfy $\mathcal{R}$. By Rule K2, k_q jobs hold ℓ_q throughout $[t_2, t_3)$. By Rule K1 and Lemma 5, each resource-holding job is always scheduled. Thus, per $L^q_{sum,h}$ time units during $[t_2, t_3)$ at least $h \cdot k_q$ requests complete—and hence, by Rule K3, at least $h \cdot k_q$ requests are dequeued from FQ_q. Dequeuing $m - k_q$ requests from FQ_q thus requires at most $L^q_{sum,\lceil \frac{m-k_q}{k_q} \rceil}$ time units, so $t_3 - t_2 \le L^q_{sum,\lceil \frac{m-k_q}{k_q} \rceil}$. ◀

Similar to the OLP-F, no release blocking occurs under the k-OLP-F. Therefore, by Lemma 13, we have the following theorem.

▶ **Theorem 14.** *Under the k-OLP-F, J_i suffers pi-blocking for at most $b_i = \sum_{q=1}^{n_r} N_i^q \cdot L^q_{sum,\lceil \frac{m-k_q}{k_q} \rceil}$ time units.*

Thus, the k-OLP-F is optimal for k-exclusion locking under C-FIFO scheduling.

7 Reader-Writer Locks

Some resources can be read without alteration. For such resources, it may be desirable to support *reader-writer* (RW) sharing. Here, *writers* have mutually exclusive access to the resource, but multiple *readers* can access the resource simultaneously.

Under RW sharing, it is often desirable to ensure fast read access. However, enabling fast read access may cause write requests to starve. This can happen under a *read-preference* RW lock that never satisfies a write request if a read request is active. More generally, these observations give rise to an important question: what is the minimum request blocking a read request can incur without causing a write request to starve?

Lower bound on read request blocking. As we show next, ensuring a read request delay of $2L^q_{max} - 2$ time units can in fact cause writer starvation.

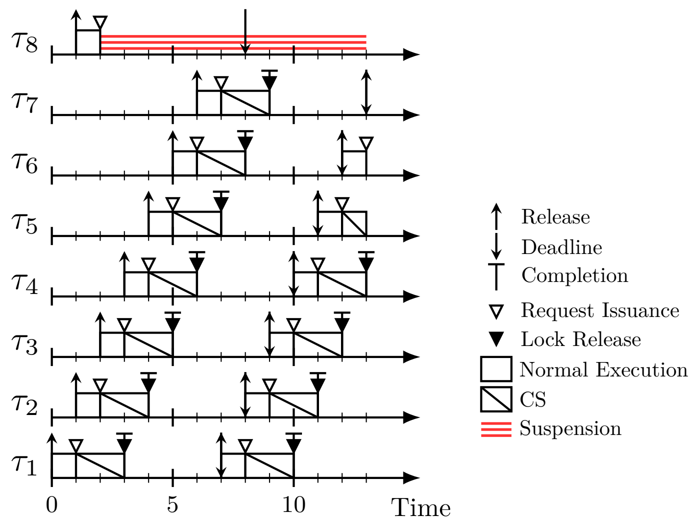


Figure 5 A schedule illustrating Theorem 15.

▶ **Theorem 15.** *For $m \geq 8$, a task system and a release sequence for it exist such that any locking protocol that ensures request blocking of at most $2L^q_{max} - 2$ time units for read requests causes unbounded request blocking for write requests under any work-conserving scheduler.*

Proof. We give an example task system Γ and a release sequence for it supporting the claim. Let $\tau_1, \tau_2, \ldots, \tau_m$ be m sporadic tasks scheduled on m processors. All tasks have WCETs of $L + 1$ time units with $2 \leq L \leq (m-2)/3$. Fig. 5 illustrates this for $m = 8$ and $L = 2$. Each job's execution consists of 1.0 time unit of non-CS execution followed by L time units of CS execution. Tasks $\tau_1, \tau_2, \ldots, \tau_{m-1}$ issue read requests for resource ℓ_q, while τ_m issues a write request for ℓ_q. The periods of all tasks are $m - 1$. Each task has an implicit deadline.

Feasibility of Γ. We show that Γ is feasible under a *write-preference* RW lock. Such lock does not satisfy any read request if a write request is waiting. Since τ_m is the only writer task, under a write-preference RW lock, τ_m's jobs acquire ℓ_q immediately (if no reader jobs hold ℓ_q) or immediately after the currently satisfied read requests complete (otherwise). Thus,

E each of τ_m's jobs acquires ℓ_q within L time units of its request issuance.

Since there are m tasks, a processor is always available for τ_m. Thus, with a WCET of $L + 1$ and resource acquisition time of at most L, each job of τ_m completes within $L + 1 + L = 2L + 1 \leq 2(m-2)/3 + 1 < m - 2 + 1 = m - 1 = T_m$ time units after its release.

For reader tasks $\tau_1, \tau_2, \ldots, \tau_{m-1}$, a read request $\mathcal{R}$ issued at time t is satisfied immediately if there is no waiting write request. Otherwise, by (E), the pending write request by τ_m's job is satisfied by time $t + L$ and complete by time $t + L + L = t + 2L$ (as a processor is available). Since τ_m is the only writer task, after completion of the write request, there is no pending write request. Thus, $\mathcal{R}$ is satisfied by time $t + 2L$. With a WCET of $L + 1$, the job issuing $\mathcal{R}$ completes within $L + 1 + 2L = 3L + 1 \leq 3(m-2)/3 + 1 = m - 2 + 1 = m - 1 = T_i$ time units after its release. Therefore, Γ is feasible.

Release sequence for Γ. τ_m releases its jobs periodically from time 1. τ_1 releases its first job at time 0 and its subsequent jobs' release times are defined as $r_{1,j+1} = f_{m-1,j} - L$. The release times of τ_i's jobs with $2 \le i < m$ are $r_{i,j} = f_{i-1,j} - L$. Thus, for $2 \le i < m$, we have

$$\begin{aligned} r_{i,j} &= f_{i-1,j} - L \\ &\ge \{\text{Since } J_{i-1,j} \text{ executes for } L+1 \text{ time units}\} \\ &\quad r_{i-1,j} + L + 1 - L \\ &= r_{i-1,j} + 1. \end{aligned} \tag{1}$$

Similarly, for τ_1, it can be shown that

$$r_{1,j+1} \ge r_{m-1,j} + 1. \tag{2}$$

We now show that consecutive jobs of τ_i with $i < m$ are released at least T_i time units apart. For $2 \le i < m$, by (1), we have

$$\begin{aligned} r_{i,j+1} &\ge r_{i-1,j+1} + 1 \\ &\ge \{\text{Applying (1) repeatedly for } i-2 \text{ times}\} \\ &\quad r_{1,j+1} + 1 + (i-2) \\ &\ge \{\text{By (2)}\} \\ &\quad r_{m-1,j} + 1 + (i-1) \\ &\ge \{\text{Applying (1) repeatedly for } m-1-i \text{ times}\} \\ &\quad r_{i,j} + (m-1-i) + i \\ &= r_{i,j} + m - 1 \\ &= r_{i,j} + T_i. \end{aligned} \tag{3}$$

Similarly, we can show that consecutive jobs of τ_1 are released at least T_1 time units apart.

We now show that each job of τ_i with $i < m$ is eligible when it is released by showing that $J_{i,j}$ completes before $J_{i,j+1}$'s release. For $2 \le i < m-1$, in the third step of the derivation of (3), applying (1) repeatedly for $m-2-i$ times instead of $m-1-i$ times, we have $r_{i,j+1} \ge r_{i+1,j} + (m-2-i) + i = r_{i+1,j} + m - 2$. Since $L \le (m-2)/3 < m-2$ and $r_{i+1,j} = f_{i,j} - L$, we get $r_{i,j+1} > r_{i+1,j} + L = f_{i,j}$. For $i = m-1$, the first step in the derivation of (3) yields $r_{m-1,j+1} \ge r_{1,j+1} + 1 + (m-1-2) = r_{1,j+1} + m - 2 > r_{1,j+1} + L$. Since $r_{1,j+1} = f_{m-1,j} - L$, we get $r_{m-1,j+1} > f_{m-1,j}$. For $i = 1$, applying (1) in (2) repeatedly for $m-3$ times, we have $r_{1,j+1} \ge r_{2,j} + m - 2 > r_{2,j} + L = f_{1,j}$. Thus, $r_{i,j+1} > f_{i,j}$ for $i < m$.

Finishing up. We now prove the theorem by showing that $J_{m,1}$'s write request is never satisfied if the request delay for read requests is at most $2L-2$. Assume that $J_{m,1}$'s request is satisfied at time t. We have $t > 2$, as $J_{m,1}$ issues its request at time 2 and $J_{1,1}$ holds ℓ_q then (under a work-conserving scheduling policy, $J_{1,1}$ acquires ℓ_q at time 1). Since the scheduling policy is work-conserving, a job $J_{i,j}$ must release ℓ_q at time t. Thus, $f_{i,j} = t$.

By the job release pattern of $\tau_1, \tau_2, \ldots, \tau_{m-1}$, there exists a job $J_{u,v}$ such that $r_{u,v} = f_{i,j} - L = t - L$. Since each job is eligible when it is released and there are m tasks, $J_{u,v}$ issues a read request $\mathcal{R}$ at time $r_{u,v} + 1 = t - L + 1 < t$ (as $L \ge 2$). Since $J_{m,1}$'s write request is satisfied at time t, $\mathcal{R}$ cannot be satisfied before time $t + L$. Since the task count is m, $J_{u,v}$ is pi-blocked for a duration of at least $t + L - (t - L + 1) = 2L - 1$ time units. Thus, request blocking for read requests exceeds $2L - 2$ time units, reaching a contradiction. ◀

Thus, read request blocking of at least $2L^q_{max} - 1$ time units is fundamental to avoid writer starvation. We now establish a lower bound on write request blocking when read

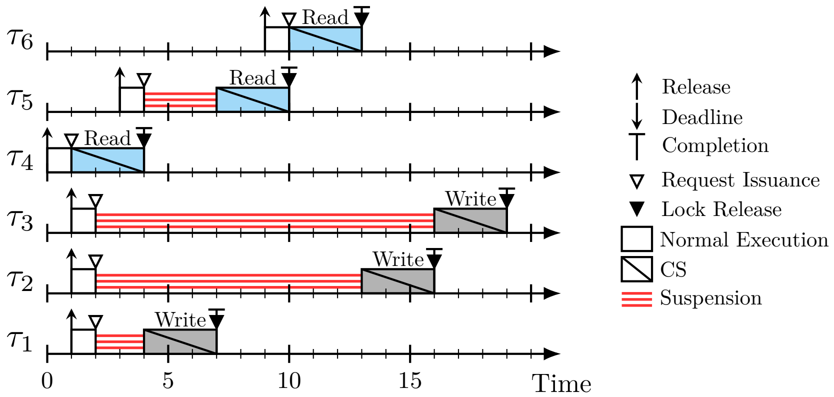


Figure 6 A schedule illustrating Theorem 16. Read and write CSs are shaded differently.

requests suffer request blocking for at most $2L^q_{max} - 1$ time units.[5]

▶ **Theorem 16.** *For $m \geq 4$, there exists a task system and a release sequence for it such that any locking protocol that ensures at most $2L^q_{max} - 1$ read request blocking causes write request blocking of $(2m-5)L^q_{max} - 1$ time units under any work-conserving scheduler.*

Proof. Let $\tau_1, \tau_2, \ldots, \tau_n$ be n tasks scheduled on $m \geq 4$ processors, where $n = 2m-4$. Each task has a WCET of $L+1$ time units with $L \geq 1$. Fig. 6 illustrates this for $m = 5$ and $L = 3$. Each job's execution consists of 1.0 time unit of non-CS execution followed by L time units of CS execution. Tasks $\tau_1, \tau_2, \ldots, \tau_{m-2}$ issue write requests for resource ℓ_q, while $\tau_{m-1}, \tau_m, \ldots, \tau_{2m-4}$ issue read requests for ℓ_q. Each task's period is $T \geq (2m-4) \cdot (L+1)$. The task WCETs sum to $(2m-4) \cdot (L+1)$, so assuming implicit deadlines, the task system can be scheduled by sequentially executing the jobs on a single processor (*i.e.*, it is feasible).

Tasks $\tau_1, \tau_2, \ldots, \tau_{m-2}$ release their first jobs at time 1. Task τ_{m-1} releases its first job at time 0. For $i > m-1$, the release time of $J_{i,1}$ is determined as $r_{i,1} = f_{i-1,1} - 1$. Hence, from time 0, there is always an eligible first job of a task until all first jobs are complete. Since all WCETs sum to $(2m-4) \cdot (L+1)$, under a work-conserving scheduler, the first job of each task completes by time $(2m-4) \cdot (L+1) \leq T$. Subsequent job release times can be easily defined so that each task's consecutive job releases are at least T time units apart.

We now prove that each first job $J_{i,1}$ always incurs pi-blocking when it is waiting for ℓ_q. For any job $J_{i,1}$ with $i > m$, we have $r_{i,1} = f_{i-1,1} - 1 \geq r_{i-1,1} + L + 1 - 1 = f_{i-2,1} - 1 + L$. Since $L \geq 1$, we have $r_{i,1} \geq f_{i-2,1}$. Thus, at most two first jobs of the last $m-2$ tasks are pending at the same time. Therefore, at most $m - 2 + 2 = m$ first jobs are pending at any time, which implies that a job $J_{i,1}$ incurs pi-blocking if it is waiting.

Finally, we prove the claim of the theorem by showing that there is a writer job that incurs pi-blocking for the duration of $(2m-5)L - 1$ time units. Job $J_{m-1,1}$ issues a read request at time 1 and acquires ℓ_q (as the scheduling policy is work-conserving). Fig. 6 illustrates this. Each job $J_{i,1}$ with $i < m-1$ issues a write request at time 2.

Each job $J_{i,1}$ with $i > m-1$ (*e.g.*, the jobs of τ_5 and τ_6 in Fig. 6) is released 1.0 time unit before $J_{i-1,1}$ completes and issues a read request when $J_{i-1,1}$ completes. Thus, $J_{i,1}$'s read request cannot be delayed to satisfy two or more pending write requests without incurring

[5] Assuming higher read request blocking would yield a smaller lower bound on write request blocking. Note that deriving tight lower bounds for RW locks is much more complicated than for the other locks considered in this paper because much leeway exists regarding the interplay between readers and writers.

Table 4 Asymptotically optimal locking protocols for RW locks under s-oblivious analysis.

Scheduling	**Protocol**	**Release blocking**	**Read request blocking**	**Write request blocking**
Clustered JLFP	CRW-OMLP [11]	$2mL_{max}$	$2L^q_{max}$	$(2m-1)L^q_{max}$
C-FIFO	RW-OLP-F (This work)	0	$2L^q_{max}-1$	$(2m-3)L^q_{max}$

read request blocking of at least $2L$ time units. As a result, at most one write request can be satisfied between two consecutive read requests. Thus, there is a write request from a job $J_{u,1}$ with $i < m-1$ (*e.g.*, τ_3's job in Fig. 6) that must be satisfied after all read and write requests of each job $J_{i,1}$ with $i \neq u$ complete.

Since $J_{u,1}$ issues its request at time 2 and $J_{m-1,1}$ (*e.g.*, τ_4's job in Fig. 6) acquires ℓ_q at time 1, $J_{m-1,1}$ pi-blocks $J_{u,1}$ for $L-1$ time units. The stated job release pattern ensures that no two of the remaining $m-3$ read requests (*e.g.*, those by τ_5 and τ_6 in Fig. 6) overlap, so they pi-block $J_{u,1}$ for $(m-3)L$ time units. Finally, $J_{u,1}$ is pi-blocked by each of the other $m-3$ write requests (*e.g.*, those by τ_1 and τ_2 in Fig. 6) for $(m-3)L$ time units. Thus, $J_{u,1}$ incurs pi-blocking for $L-1+(m-3)L+(m-3)L=(2m-5)L-1$ time units. ◀

For simplicity, Theorems 5 and 16 are stated for work-conserving scheduling. However, both theorems are also true under a wider class of schedulers and locking protocols that are *top-c-work-conserving*. On a c-processor cluster, a top-c-work-conserving scheduling ensures that any top-c-highest priority ready job immediately acquires a shared resource (including processor) if such a resource is idle. Note that a work-conserving scheduler and locking protocol combination is also top-c-work-conserving.

Asymptotically optimal RW locking protocols. For RW locks, the CRW-OMLP is an asymptotically optimal locking protocol under clustered JLFP scheduling [11]. The CRW-OMLP is a *phase-fair* RW locking protocol. *Phase-fair* RW locks satisfy read and write requests in alternating phases [12]. At the beginning of a *reader phase*, all waiting read requests are satisfied simultaneously, while at the beginning of a *writer phase*, a single waiting write request is satisfied. Tbl. 4 summarizes the CRW-OMLP.

The RW-OLP-F. We now introduce the *read-optimal RW locking protocol under C-FIFO scheduling (RW-OLP-F)*, which achieves optimal pi-blocking for read requests under C-FIFO scheduling. The RW-OLP-F is a phase-fair RW locking protocol that achieves $2L^q_{max}-1$ (resp., $(2m-3)L^q_{max}$) request blocking for read (resp., write) requests—here, however, we only prove a bound of $2L^q_{max}$ for reads due to space limitation. Unlike the CRW-OMLP, the RW-OLP-F has no release blocking under C-FIFO scheduling.

Structures. For each resource ℓ_q, we have two queues RQ^1_q and RQ^2_q that contain read requests for ℓ_q, and a FIFO queue WQ_q that contains write requests for ℓ_q. One of the read queues acts as a *collecting* queue and the other acts as a *draining* queue. The roles of RQ^1_q and RQ^2_q alternate, *i.e.*, each switches over time between being the collecting queue and being the draining queue. Initially, RQ^1_q is the collecting queue and RQ^2_q is the draining queue.

Reader rules. Assume that a job J_i attempts to issue a read request $\mathcal{R}$ for resource ℓ_q. Let RQ^c_q and RQ^d_q be the collecting and draining queues, respectively, when J_i issues $\mathcal{R}$.

R1 J_i is allowed to issue $\mathcal{R}$ only if it is one of the c highest-priority eligible jobs in its cluster. J_i suspends if necessary to ensure this condition.

R2 If WQ_q is empty when J_i issues $\mathcal{R}$, then $\mathcal{R}$ is immediately satisfied and enqueued in RQ^d_q. Otherwise, J_i suspends and $\mathcal{R}$ is enqueued in RQ^c_q.

R3 If $\mathcal{R}$ is in RQ_q^c, then it is satisfied (along with all other requests in RQ_q^c) when RQ_q^c becomes the draining queue (see Rule W3). If RQ_q^c becomes the draining queue at time t and a read request is issued at time t, then that request is enqueued in RQ_q^c before making it the draining queue. $\mathcal{R}$ is removed from RQ_q^c when it is complete. If RQ_q^c becomes empty because of $\mathcal{R}$'s removal, then the head of WQ_q (if any) is satisfied.

Writer rules. When a job J_w attempts to issue a write request $\mathcal{R}$ for a resource ℓ_q, it proceeds according to the following rules.

W1 J_w is allowed to issue $\mathcal{R}$ only if it is one of the c highest-priority eligible jobs in its cluster. J_w suspends if necessary to ensure this condition.

W2 If RQ_q^1, RQ_q^2, and WQ_q are empty when $\mathcal{R}$ is issued, then $\mathcal{R}$ is immediately satisfied and enqueued in WQ_q. Otherwise, $\mathcal{R}$ is enqueued in WQ_q and J_w suspends.

W3 Let RQ_q^d and RQ_q^c be the draining and collecting queues, respectively, when $\mathcal{R}$ is the head of WQ_q. $\mathcal{R}$ is satisfied when $\mathcal{R}$ is the head of WQ_q and RQ_q^d is empty. When $\mathcal{R}$ is complete, $\mathcal{R}$ is dequeued from WQ_q and if RQ_q^c is non-empty, then RQ_q^c (resp., RQ_q^d) becomes the draining (resp., collecting) queue. Otherwise (RQ_q^c is empty), the new head of WQ_q (if any) is satisfied.

Analysis. We now determine an upper bound on request blocking. For $m \leq 2$, by Lemma 4 and Rules R1 and W1, there are at most two active requests and at most one waiting request at any time, so request blocking is at most L^q_{max} time units for both reads and writes. Henceforth, we assume $m \geq 3$. The following lemma follows from Lemma 4 and Rules R1 and W1; we omit its proof as it is similar to Lemma 7.

▶ **Lemma 17.** *The total number of requests in RQ_q^1, RQ_q^2, and WQ_q is at most m.*

We now give two helper lemmas.

▶ **Lemma 18.** *If a write request $\mathcal{R}$ is the head of WQ_q at time t, then it is satisfied by time $t + L^q_{max}$.*

Proof. Let RQ_q^c and RQ_q^d be the collecting and draining queue, respectively, at time t. If $\mathcal{R}$ is not satisfied at time t, then by Rule W3, RQ_q^d is non-empty at time t. By Rule R3, jobs with requests in RQ_q^d hold ℓ_q at time t. Let t' be the time instant when all such requests are complete. By Lemma 5 and Rule R1, $t' \leq t + L^q_{max}$. By Rule R2, no read requests are enqueued in RQ_q^d during $[t, t')$. Thus, RQ_q^d becomes empty at time t'. By Rule W3, $\mathcal{R}$ is satisfied at time t'. Thus, the lemma holds. ◀

▶ **Lemma 19.** *If a write request $\mathcal{R}$ is the head of WQ_q at time t, then it is complete by time $t + 2L^q_{max}$.*

Proof. By Lemma 18, $\mathcal{R}$ is satisfied by time $t + L^q_{max}$. By Lemma 5 and Rule W1, $\mathcal{R}$ completes within L^q_{max} time units after being satisfied. Thus, the lemma holds. ◀

We now determine an upper bound on the request blocking suffered by a job when it issues a read request. We consider a job J_i that issues a read request $\mathcal{R}$ for resource ℓ_q. As depicted in Fig. 3, let t_1, t_2, t_3, and t_4 be the time instants corresponding to when J_i attempts to issue $\mathcal{R}$, and when $\mathcal{R}$ is issued, satisfied, and complete, respectively. In the lemma below, for simplicity, we show that request blocking for read requests is at most $2L^q_{max}$. A tight bound of $2L^q_{max} - 1$ can be established by a detailed analysis involving multiple cases.

▶ **Lemma 20.** *For a read request $\mathcal{R}$, J_i suffers request blocking for at most $2L^q_{max}$ time units.*

Proof. J_i suffers pi-blocking for the duration of $[t_2, t_3)$. Let RQ^c_q and RQ^d_q be the collecting and draining queue, respectively, at time t_2. If WQ_q is empty at time t_2, then $t_2 = t_3$ holds according to Rule R2, so assume otherwise. By Rule R2, $\mathcal{R}$ is enqueued in RQ^c_q. Let $\mathcal{R}'$ be the request at the head of WQ_q at time t_2. Let t'_2 be the time instant when $\mathcal{R}'$ completes. By Lemma 19, $t'_2 \leq t_2 + 2L^q_{max}$ holds. By Rule W3, RQ^c_q becomes the draining queue at time t'_2. Thus, by Rule R3, all requests in RQ^c_q, including $\mathcal{R}$, are satisfied at time t'_2, implying $t_3 = t'_2$. Therefore, we have $t_3 - t_2 \leq 2L^q_{max}$. ◀

Finally, we give an upper bound on the request blocking incurred by a job when issuing a write request. Let J_w be a job that issues a write request $\mathcal{R}$ at time t.

▶ **Lemma 21.** *For a write request $\mathcal{R}$, J_w incurs request blocking for at most $(2m-3)L^q_{max}$ time units.*

Proof. If no request holds ℓ_q at time t, then by Rule W2, $\mathcal{R}$ is immediately satisfied. This leaves two cases.

Case 1. *A job with a read request holds ℓ_q at time t.* By Lemma 17, RQ^1_q, RQ^2_q, and WQ_q hold at most m requests at time t. Since there is an active read request, at most $m-2$ write requests precede $\mathcal{R}$ in WQ_q. By Rule W3, each of those write requests becomes the head of WQ_q when its preceding write request completes. By Lemma 19, a write request at the head of WQ_q completes within $2L^q_{max}$ time units from when it becomes the head. Thus, all $m-2$ write requests that precede $\mathcal{R}$ in WQ_q are complete by time $t + 2(m-2)L^q_{max}$. By Lemma 18, after becoming the head of WQ_q, $\mathcal{R}$ is satisfied within an additional L^q_{max} time units. Thus, $\mathcal{R}$ is satisfied by time $t + (2m-3)L^q_{max}$.

Case 2. *A job with a write request $\mathcal{R}'$ holds ℓ_q at time t.* We consider two subcases.

Case 2a. *WQ_q contains m requests at time t.* Thus, $m-1$ requests precede $\mathcal{R}$ in WQ_q. By Lemma 5 and Rule W1, $\mathcal{R}'$ completes within L^q_{max} time units from t. By Lemma 4 and Rules R1 and W1, no requests are issued before $\mathcal{R}'$ completes. Thus, by Rule W3, the write request $\mathcal{R}''$ following $\mathcal{R}'$ is satisfied when $\mathcal{R}'$ is complete. By Lemma 5 and Rule W1, $\mathcal{R}''$ completes within L^q_{max} time from when it is satisfied. Thus, the top two requests in WQ_q are complete by time $t + 2L^q_{max}$. By Lemma 19, each of the remaining $m-3$ write requests preceding $\mathcal{R}$ is complete within $2L^q_{max}$ time units after becoming the head of WQ_q. Thus, $\mathcal{R}$ becomes the head of WQ_q by time $t + 2L^q_{max} + 2(m-3)L^q_{max} = t + 2(m-2)L^q_{max}$. By Lemma 18, $\mathcal{R}$ is satisfied within L^q_{max} time units after becoming WQ_q's head. Thus, $\mathcal{R}$ is satisfied by time $t + (2m-3)L^q_{max}$.

Case 2b. *WQ_q contains at most $m-1$ requests at time t.* Thus, at most $m-2$ requests precede $\mathcal{R}$ in WQ_q. By Lemma 5, $\mathcal{R}'$ completes within L^q_{max} time units from t. By Lemma 19, each of the remaining $m-3$ write requests preceding $\mathcal{R}'$ completes within $2L^q_{max}$ time units from when it becomes the head of WQ_q. Thus, $\mathcal{R}$ becomes the head of WQ_q within $L^q_{max} + 2(m-3)L^q_{max} = (2m-5)L^q_{max}$ time units from t. By Lemma 18, $\mathcal{R}$ is satisfied within L^q_{max} time units after becoming WQ_q's head. Thus, $\mathcal{R}$ is satisfied by time $(2m-4)L^q_{max}$. ◀

Similar to the OLP-F, no job suffers release blocking due to a resource-holding job under the RW-OLP-F. By Lemma 20 and 21 and letting $N^{q,r}_i$ and $N^{q,w}_i$ denote the maximum number of read and write requests for ℓ_q by τ_i, we have the following.

▶ **Theorem 22.** *Under the RW-OLP-F, J_i is pi-blocked for at most*

$$b_i = \sum_{q=1}^{n_r} (N_i^{q,r} \cdot 2L_{max}^q + N_i^{q,w} \cdot (2m-3)L_{max}^q).$$

As mentioned already, the $2L_{max}^q$ term above can be replaced by $2L_{max}^q - 1$ at the expense of more lengthy analysis. By Rules R1, R2, W1, and W2, FIFO scheduling and RW-OLP-F ensures top-c-work-conserving property. Thus, by Theorems 15 and 16, the RW-OLP-F ensures optimal request blocking for read requests, while ensuring that the request blocking for write requests is just under two request lengths of optimal.

8 Experimental Evaluation

In this section, we present the results of experiments we have conducted using the SchedCAT toolkit [1] to evaluate our proposed locking protocols.

Task system generation. Our task-system generation method is similar to that used in prior locking-related schedulability studies [6, 9, 32]. We generated task systems randomly for systems with $\{4, 8, 16\}$ processors. For each processor count, we generated task systems that have a *normalized utilization*, *i.e.*, $\sum_{i=1}^{n} u_i/m$, from 0.2 to 0.9 with a step size of 0.1. We chose the number of tasks uniformly from $[2m, 150]$. We generated each task's utilization uniformly following procedures from [19]. We chose each task's period randomly from $[3, 33]$ms (*short*), $[10, 100]$ms (*moderate*), or $[50, 500]$ms (*long*). We set each task's WCET C_i to $T_i \cdot u_i$ rounded to the next microsecond.

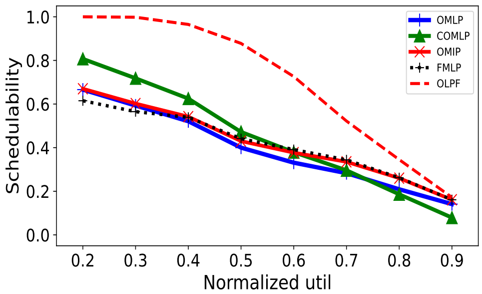
We considered $\{m/4, m/2, m, 2m\}$ number of shared resources. For each τ_i and resource ℓ_q, we selected τ_i to access resource ℓ_q with probability $p^{acc} \in \{0.1, 0.25, 0.5\}$. If so selected, τ_i was defined to access ℓ_q via $N_i^q \in \{1, 2, \ldots, 5\}$ requests. For each $N_i^q > 0$, we chose the maximum request length L_i^q randomly from three uniform distributions ranging over $[1, 15]\mu$s (*short*), $[1, 100]\mu$s (*medium*), or $[5, 1280]\mu$s (*long*). A chosen L_i^q value was decreased accordingly if it caused the sum of all request length of τ_i to exceed C_i. For each combination of m, normalized utilization, T_i, L_i^q, p^{acc}, and n_r, we generated 1,000 task systems. We call each combination of these parameters a *scenario*.

Experiment 1. In our first experiment, we considered mutex sharing. Each task had a *soft* timing constraint, meaning that it was deemed schedulable if its response time was bounded. We considered resource synchronization under the OLP-F, the OMLP [11], the C-OMLP [13], the OMIP [7], and the FMLP [5]. For the OLP-F, each task system's schedulability was tested under global FIFO scheduling [22]. For the remaining protocols, s-oblivious schedulability tests were performed under global EDF scheduling [16].[6] For each scenario, we assessed *acceptance ratios*, which give the percentage of task systems that were schedulable under each locking protocol. We present a representative selection of our results in Fig. 7.

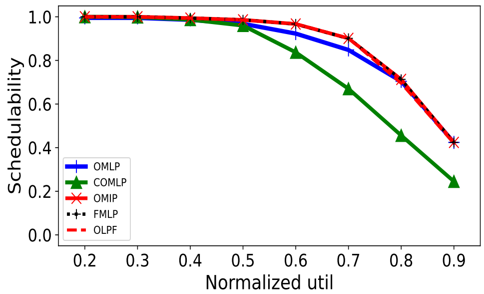
▶ **Observation 1.** *The average improvement under the OLP-F over the OMLP, the C-OMLP, the OMIP, and the FMLP was 20.2%, 14.9%, 16.4%, and 27.5%, respectively.*

This can be seen in insets (a) and (b) of Fig. 7. Unsurprisingly, schedulability was improved under the OLP-F because of lower pi-blocking compared to the other protocols. In some cases, as depicted in Fig. 7(b), all protocols had similar schedulability. This can

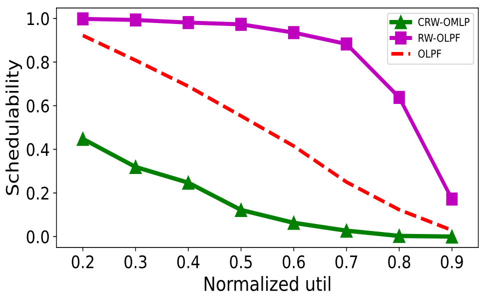
[6] The same schedulability test also applies for a wider class of global schedulers including FIFO.



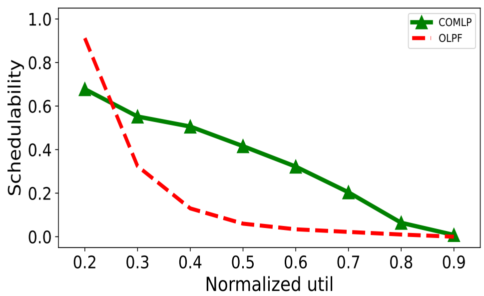
(a) Exp. 1 with $m = 8$, moderate periods, medium requests, $p^{acc} = 0.2$, $n_r = 2$.



(b) Exp. 1 with $m = 16$, short periods, short requests, $p^{acc} = 0.1$, $n_r = 8$.



(c) Exp. 2 with $m = 8$, long periods, medium requests, $p^{acc} = 0.2$, $p^{write} = 0.3$, $n_r = 32$.



(d) Exp. 3 with $m = 4$, long periods, short requests, $p^{acc} = 0.5$, $n_r = 8$.

Figure 7 Experimental results.

happen when the number of request-issuing jobs for each resource is small (*e.g.*, less than the number of processors), in which case all protocols have similar pi-blocking bounds.

Experiment 2. This experiment pertains to RW sharing. To generate task systems, we used one additional parameter $p^{write} \in \{0.1, 0.2, 0.3, 0.5, 0.7\}$. We defined each resource access to be a write (resp., read) access with probability p^{write} (resp., $1 - p^{write}$). In this experiment, we considered soft real-time scheduling with resource synchronization under the RW-OLP-F, the CRW-OMLP [13], and the OLP-F. Each task system's schedulability was tested under global FIFO scheduling when the OLP-F and the RW-OLP-F were employed, and under global EDF scheduling otherwise. We have the following observation.

▶ **Observation 2.** *The RW-OLP-F improved schedulability over the CRW-OMLP across all scenarios. The RW-OLP-F had less schedulability than the OLP-F when write accesses were more frequent, i.e., high p^{write} values.*

This can be seen in Fig. 7(c). The improved pi-blocking bound enabled higher schedulability under the RW-OLP-F. The RW-OLP-F had better or equal schedulability than the OLP-F across 90% of the total scenarios. Since the RW-OLP-F has higher write request blocking compared to the OLP-F (which does not have optimal read request blocking), the OLP-F had better schedulability than the RW-OLP-F when p^{write} values are high, *e.g.*, $p^{write} = 0.7$.

Experiment 3. In this experiment, we considered hard real-time scheduling under mutex locks. For each task τ_i, we randomly chose a relative deadline between $[T_i, 2T_i]$. We considered partitioned scheduling because of the lack of hard real-time schedulability tests for global

FIFO scheduling. We used the *worst-fit* bin packing heuristic to partition each task system. We compared schedulability under the OLP-F and partitioned FIFO scheduling with the partitioned OMLP (the C-OMLP with $c = 1$) and partitioned EDF scheduling.

▶ **Observation 3.** *The partitioned OMLP had better schedulability compared to the OLP-F.*

This can be seen in Fig. 7(d). Despite having lower pi-blocking and bounded response times, the partitioned OMLP enabled better schedulability because of the optimality of uniprocessor EDF in scheduling hard real-time workloads. Note that, unlike for EDF, the employed FIFO schedulability test was non-exact [4].

9 Related Work

The literature on suspension-based multiprocessor real-time locking protocols is quite vast (*e.g.*, [7, 11, 13–15, 17, 20, 21, 23–25, 27, 29]). An excellent recent survey is given in [10]. Below, we comment further on a few specific relevant protocols.

In work on mutex locks, the FMLP [5] was the first multiprocessor locking protocol to be studied under s-oblivious analysis. While relatively simple, the FMLP has $O(n)$ pi-blocking under s-oblivious analysis. The first mutex protocols that were shown to have asymptotically optimal s-oblivious pi-blocking were the OMLP and its variants, which include protocols applicable under partitioned, global, and clustered JLFP scheduling [11, 13, 14]. In later work, the OMIP [7] was presented; it upholds an *independence preserving* property that results in asymptotically optimal s-oblivious pi-blocking under clustered JLFP scheduling.

The first multiprocessor mutex locking protocols were designed to be studied under s-aware analysis. Many of these protocols (*e.g.*, the MPCP [27], the PPCP [17], the PIP [26], *etc.*) were inspired by classical uniprocessor locking protocols. The FMLP$^+$ [9] is an extension of the FMLP that has been shown to have asymptotically optimal s-aware pi-blocking under clustered JLFP scheduling. In other work, linear-programming techniques were proposed that enable improved s-aware analysis of various protocols, including the PIP, the PPCP, and the FMLP, under global and partitioned fixed-priority scheduling [8, 32].

10 Conclusion

In this paper, we have presented optimal suspension-based multiprocessor locking protocols for mutex, k-exclusion, and RW synchronization. In particular, we have shown that the s-oblivious lower bound of $m - 1$ request lengths for mutex locks is indeed tight under FIFO scheduling. We have also provided a tight s-oblivious lower bound on read-request blocking for RW locks. All three locking protocols presented herein can be used together in the same system without jeopardizing the presented analysis. Moreover, spin-based versions of these protocols can be easily obtained by following the same design principles.

For some non-FIFO JLFP schedulers, it may be possible that $2m - 1$ request lengths is indeed a tight lower bound on s-oblivious pi-blocking for mutex locks. Showing this would require a new lower-bound proof. As seen in Sec. 7, finding task systems that justify such a lower bound can be quite difficult. The results of this paper show that any task system used to justify a $2m - 1$ lower bound must necessarily not be FIFO-scheduled. In some sense, this is unfortunate, as FIFO schedules are somewhat easier to deal with in lower-bound arguments, given that having "top-c" priority is a stable property for FIFO-scheduled jobs.

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