More Shape Representations

- Currents
  - Representations via tangents
  - Deformetrica, using Currents
- Signed distance functions
  - NeRFs and visualizations via them

Curves in Deformetrica (Currents)

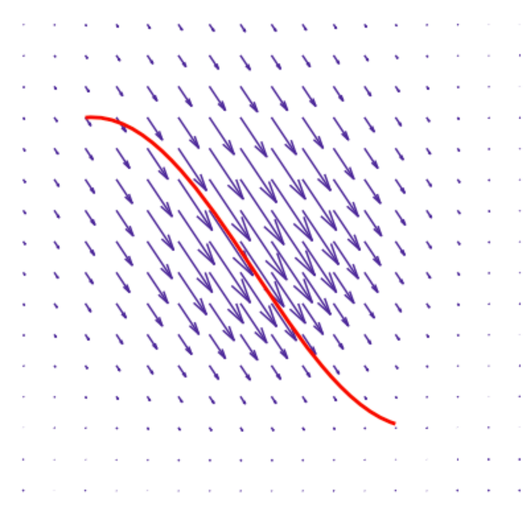
- J Glaunès, A Qiu, ML Miller, Younes. Large Deformation Diffeometric Metric Curve Mapping. IJCV, 2008.

- Distance between shape C and shape S
  - $$J_{C,S}\left((v_t)_{t\in[0,1]}\right) \doteq \gamma \int_0^1 \|v_t\|_V^2\, dt + E(\phi_1^v \cdot C, S),$$
  - Minimized via LDDMM
- Points with curve tangents
  - Better than point-to-point matching alone
  - Based on reproducing kernels
  - Works when data has noise

Curves in Deformetrica (Currents), 2

- Curve matching to an image **w** of vectors:

$$\langle \mu_C | w \rangle \doteq \int_0^1 \gamma'_C(s) \cdot w(\gamma_C(s))ds.$$

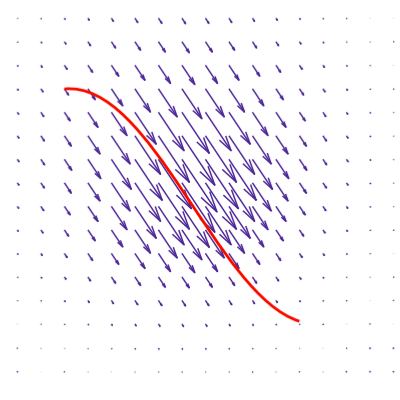


  - Via agreement of tangents of interrogated curve
  - Captures both position and direction
- So matching curve C with curve S
  - µC|**w** difference metric to µS|**w** small
    - What **w**? Extrapolation of the other
      - Based on reproducing kernels
      - Typically Gaussian
      - ?? Strong curvatures handled poorly

$$\int_0^1 \gamma'_S(s) \cdot w(\gamma_S(s)) - \gamma'_C(s) \cdot w(\gamma_C(s))\, ds.$$

Curves in Deformetrica (Currents), 3

- Curve positions and tangents matching to an image **w** of vectors:



$$\langle \mu_C | w \rangle \doteq \int_0^1 \gamma_C'(s) \cdot w(\gamma_C(s)) ds,$$

- Deform C to S

$$E(\phi(C), S) \doteq \left\| \mu_{\phi(C)} - \mu_S \right\|_{W^*}^2$$

  - Via LDDMM, producing velocities
- Matching deformed curve C with curve S
  - Via difference metric:

$$\left| \int_0^1 \gamma_S'(s) \cdot w(\gamma_S(s)) - \gamma_C'(s) \cdot w(\gamma_C(s))\, ds. \right|$$

    - What **w**?
      - Based on reproducing kernels

Signed Distance Image (SDI) as Shape Representation

- Distance to boundary, with sign indicating interior vs. exterior
  - Boundary is zero level surface of SDI
  - Only really zero order, but related to Blum measures
  - Crossing problems in exterior
- Used in turning binary images of voxels into a boundary mesh (Marching Cubes)
- Used to produce boundary evolutions, incl. smoothings, that allow topological changes
- Used to produce Neural Radiance Functions (NeRFs), ~opacities, that can be used to produce boundary visualizations using Generative Neural Nets
  - Trained from sparse collection of poses

NeRF: Representing Scenes as Neural Radiance Fields for View Synthesis,
Ben Mildenhall, *Pratul Srinivasan*, Matthew Tancik*, Jonathan Barron, Ravi Ramamoorthi, Ren Ng, ECCV 2020.