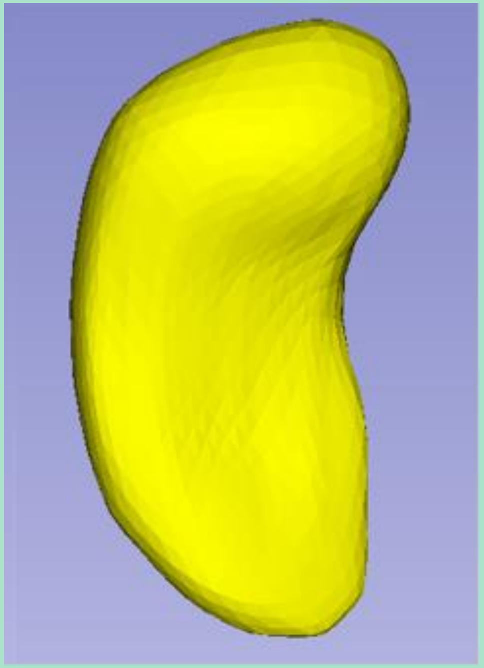
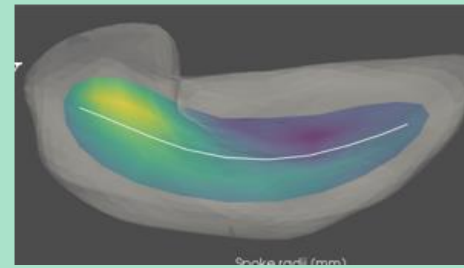
Shape Representations and Statistics
Course Overview

Stephen Pizer

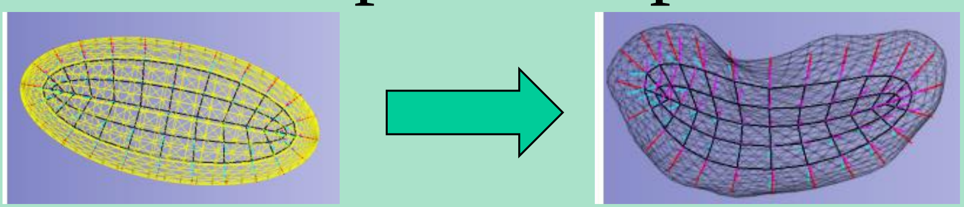


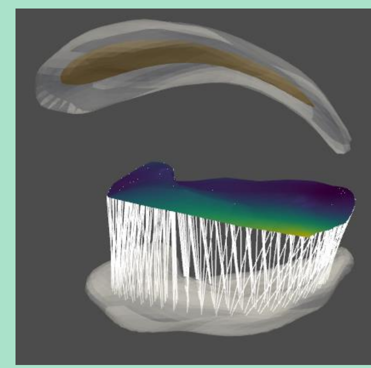
Course Objectives

- Briefly present background math and stats
- Survey shape representations

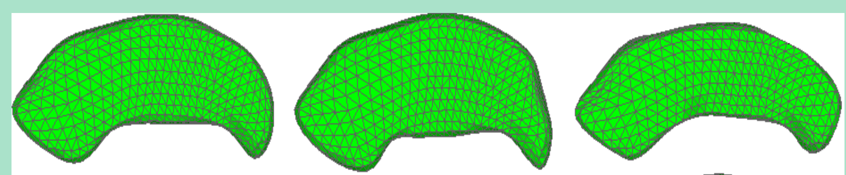


  - Boundary representations
  - Skeletal representations
  - Diffeomorphism representations





  - Multi-object representations
- Cover statistics of shape methods
  - Statistics objectives
  - Statistics on manifolds



Where and when to Participate

- Times: T Th 3:30-4:45, Pizer office hours TBA
  - Appts
    - Set up via pizer@cs.unc.edu or at 919 590 6085
    - In SN 222 or on class zoom site
- Access
  - In class: FB 007
  - Zoom: https://unc.zoom.us/j/94989560766 passcode:205403
  - Recordings
    - Catalogue and youtube addresses: on website cs.unc.edu/shape-comp-790-6
    - Lectures on youtube
      - Including ppts, board contents, questions and answers
- For credit as UNC student or audit
  - UNC students could do an audit for credit
- **Every participator should email pizer@cs.unc.edu**

Background Assumed

- Multi-dimensional calculus
  - Gradient, directional derivatives
  - Hessian (2nd derivative matrix), Laplacian
- Linear algebra
  - Matrices as linear transformations, eigenvectors & values
- 1 course in mathematical probability
  - Probability densities, joint densities
  - Conditional probability densities, Bayes rule
  - Normal (Gaussian) probability distribution

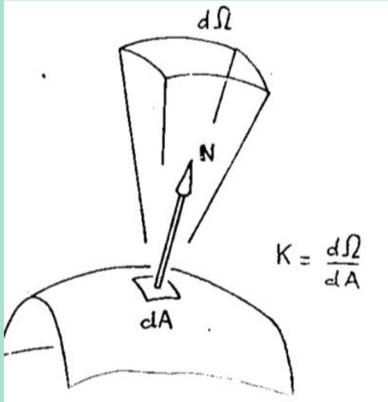
How to Participate

- Ask lots of questions
  - Whether you are taking for credit or audit
  - Whether you are attending in person or via zoom
    - Via zoom, interrupt or raise virtual hand
- Keep up with the lectures
  - If you miss, look at recording immediately
  - For those taking for credit, and ideally for all, do the readings in a timely fashion
- For those taking the course for credit
  - Choose a project during first month, based on a meeting with Prof. Pizer
    - You will present your project orally and in writing at the end of the course
  - Expect an oral final exam

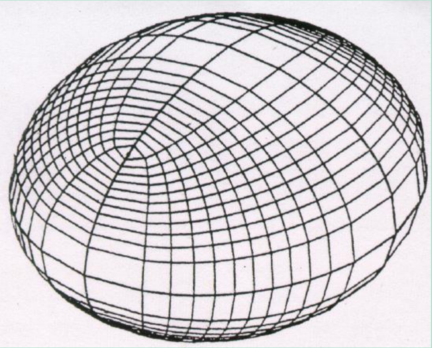
The Readings

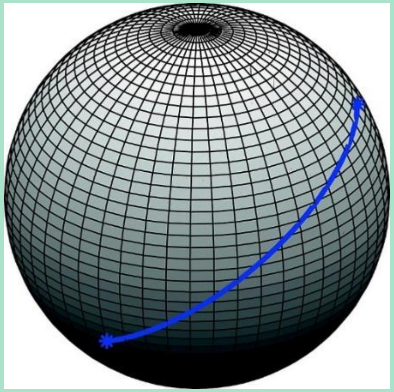
- In 4 books on reserve (or buy your own)
  - JS Marron and IL Dryden, *Object Oriented Data Analysis*, CRC Press.2022
  - X Pennec, S Sommer, T Fletcher, *Riemannian Geometric Statistics in Medical Image Analysis*, Academic Press, 2020
  - G Zheng, S Li, G Szekely, *Statistical Shape and Deformation Analysis*, Academic Press, 2017,
  - K Siddiqi and S Pizer, *Medial Representations: Mathematics, Algorithms and Applications*, Springer 2008
- In many papers, on Pizer's google drive

Shape Representation Geometry and Topology



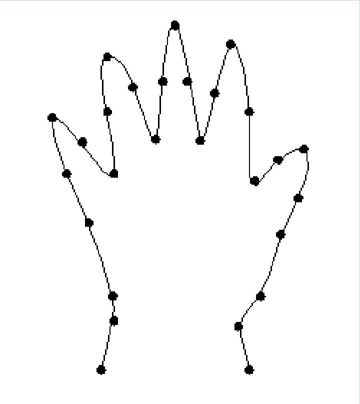
- Normal directions and tangent directions
- Curvatures: curves and surfaces
- Number of holes & connected components topology
- Shape spaces
- Manifolds and geodesics
- Distance measures
  - Riemannian metrics

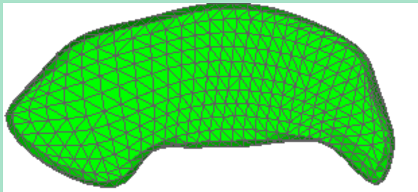


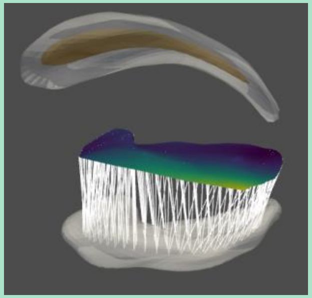


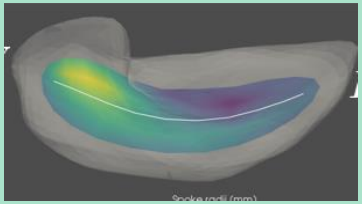
Shape Representation Categories

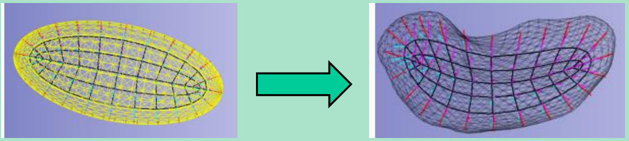
- Landmarks
- Objects
  - Boundaries
    - Points
    - Meshes
    - Normals
    - Spherical harmonics, Fourier
    - Signed distance images
  - Interiors and interior algebraic graphs
  - Skeletal models
  - Landcurves: currents
  - Multi-object representations
- Diffeos from a central example



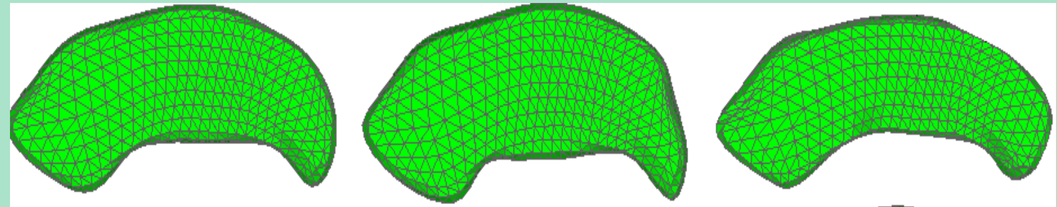




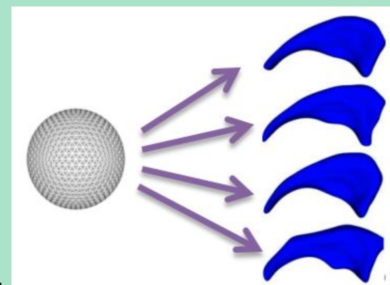


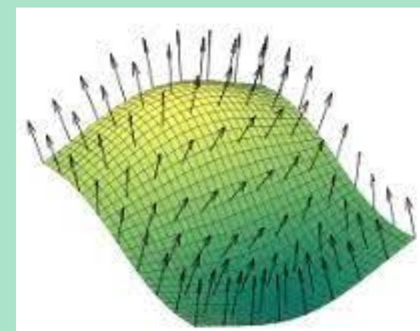


Shape Representation by Boundary Points



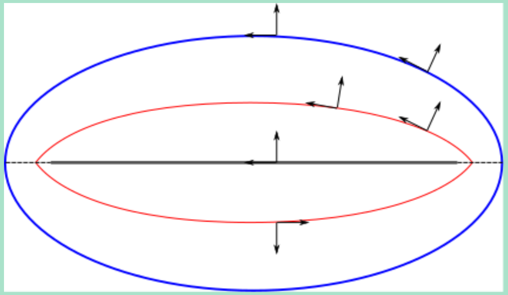
- Points in correspondence (PDM); or Meshes
  - Correspondence produced by
    - Diffeomorphisms
    - Skeletal models
    - Entropy minimization



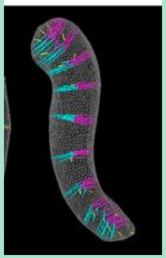


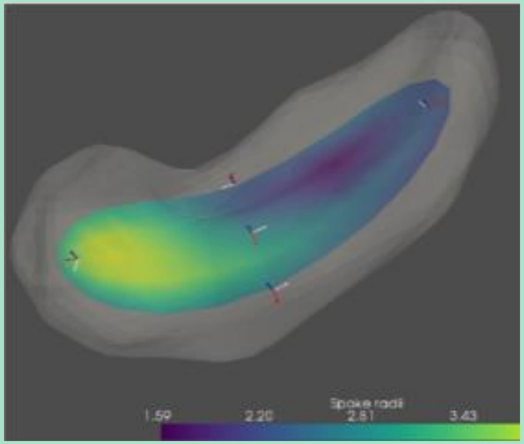
- Spherical harmonics, Fourier
- Points with normal, normals alone
- Points with tangents on landcurves (Currents)
- Normals with correspondence mod-ed out
- Signed distance images
- Distance measures
  - Riemannian metrics

Shape Representation by Skeletal Models



- Medial and skeletal mathematics
- S-reps
  - Skeletal points, spokes
  - Diffeomorphisms from ellipsoids, fitted frames
  - Taheri via swept planar cross-sections
  - Fitting to boundaries
    - Optimization
    - CNN

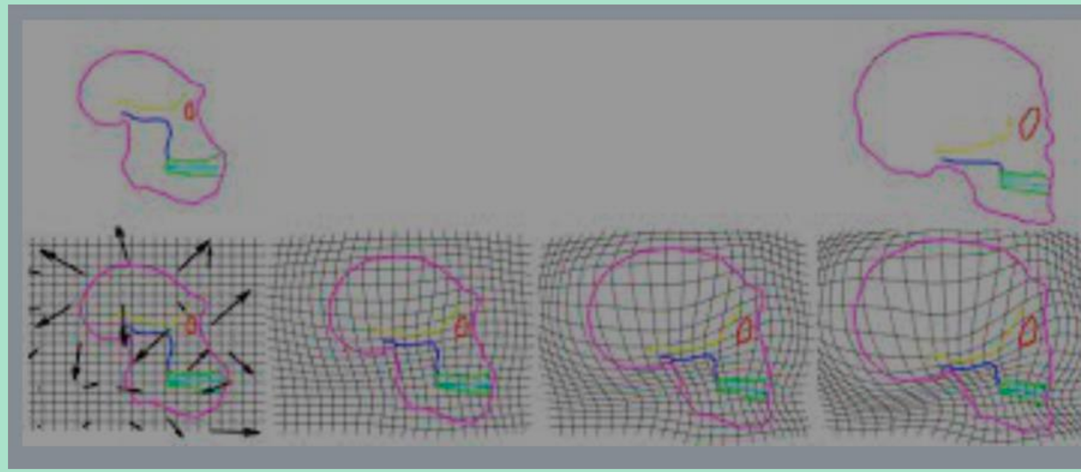


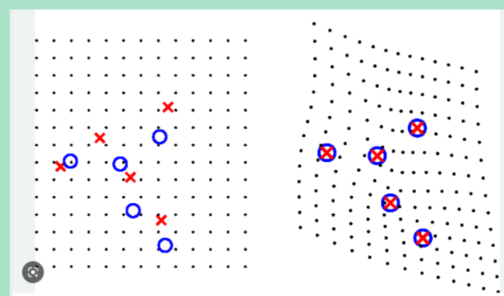


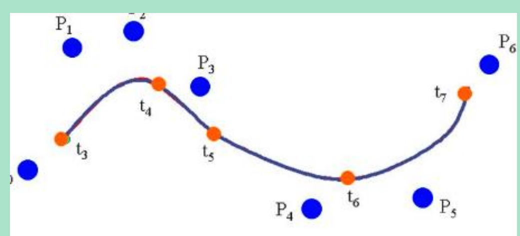
- Cm-reps [2 lectures by P. Yushkevich]
  - Based on PDE
  - Based on splines in $\underline{x}$ and half-width r
  - Fitting to boundaries

Shape Representation by Deformations

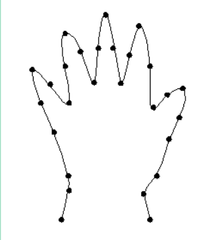
- Diffeomorphisms: velocities
  - Points data
  - Currents data
    - For landcurves
    - For surfaces
  - Using CNN
- Displacements
  - Thin-plate splines
  - B-splines
  - Elastic deformations



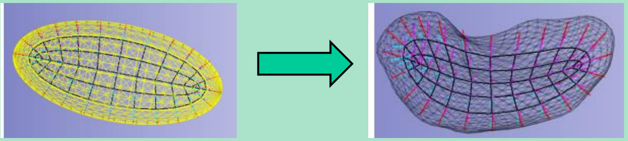


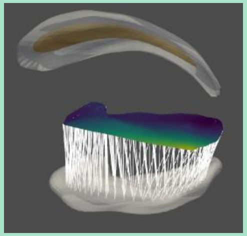


Shape Statistics Features



- Correspondence
- Point lists (PDMs in correspondence)
- Coefficients of orthogonal bases
  - Spherical harmonics, Fourier
  - PCA bases
- Directions
  - Normals, displacements, velocities
  - Spokes in skeletal models
  - Inter-object links





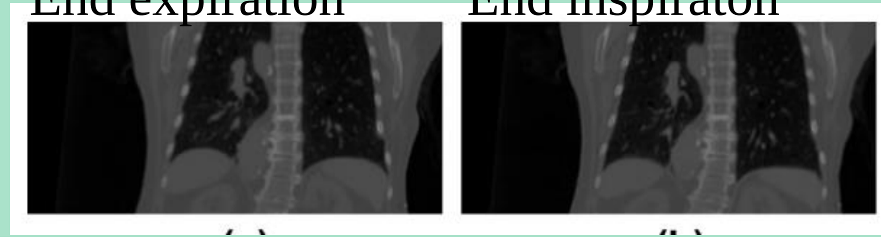
- Scalars: volume, width, length
- Momenta of diffeos ~ Initial geodesic direction
- Derived by PCA-like methods: PCA, PGA, PNS

Shape Statistics Objectives

Non-Euclidean features

- Classification
- Hypothesis testing
  - Permutation tests (distribution-free)
- Segmentation
  - Appearance: geometry-relative intensity features
  - Geometric features (e.g., anatomy)
- Temporal processes
  - Deformations
  - Longitudinal

End expiration End inspiraton

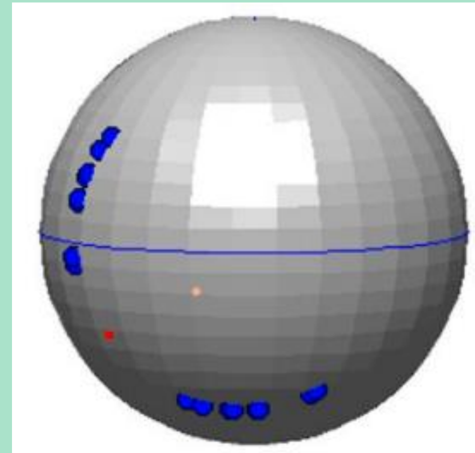


Shape Statistics Preprocessing

- Commensuration: scaling and weighting
- Euclideanization
  - Positive scalars
  - Directions
  - Normalized PDMs
  - Local charts

Shape Statistics Components

- Means
  - Fréchet
  - Backwards; Barycentric
  - Extrinsic
  - Diffusion
- Modes of variation
  - Forward
  - Backwards
- Covariance and correlation; entropy
- Shape on Riemannian manifolds [Fletcher]
- Geodesics
  - Of diffeomorphisms
  - Probabilistic geodesics [Sommer]



Backwards PNS mean (red dot) vs. Fréchet (light dot) mean

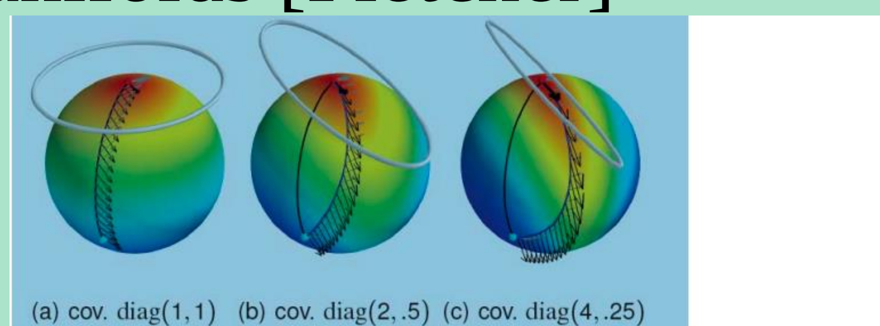
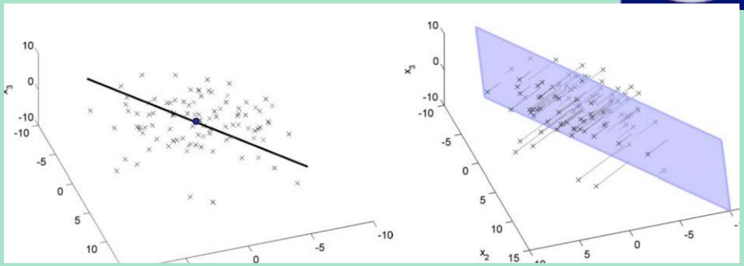
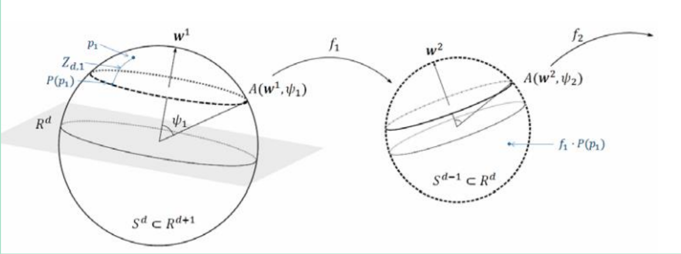


Figure 10. Most probable paths on a sphere with different Brownian covariances.

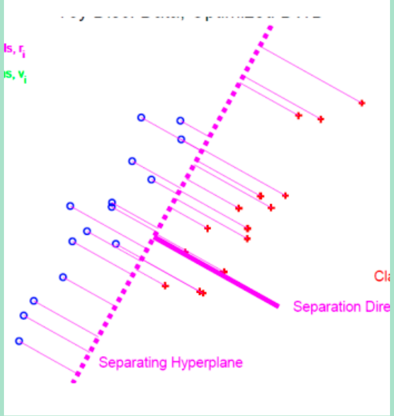
Shape Statistics Methods



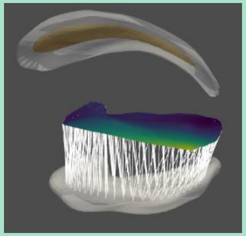
- Principal Component Analysis
- Principal Nested Spheres



- Principal Geodesic Analysis
- Correspondence via entropy
- SVM and DWD for separation directions
  - Kernels



- Within and between entities: AJIVE
- CNNs
  - Classification
  - NeRFs (Neural Radiance Fields)
    - Production by optimization
    - Rendering



Shape Statistics Effectiveness Measures

- Generalizability, Specificity
- Cross-validation
- AUC (and ROC) for classification
- DiProPerm
- Applications
  - Objectives and methods
  - Measures of success

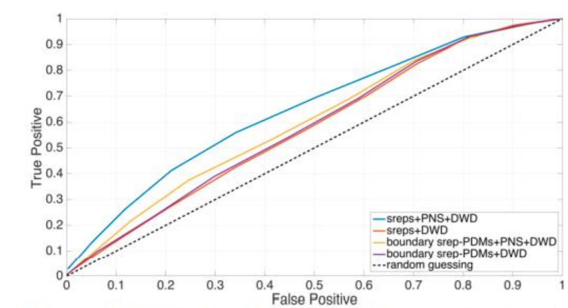


Figure15. ROCs for classifying hippocampi as to sch

Course Order: First Part

- Overview of course
- Introduction to shape representations
- Mathematics background
  - Curved surface geometry, topology, ridges, Riemannian manifolds
- PDMs
  - Statistics on PDMs
    - PCA (both directions), PNS
    - Kendall shape space
- Spherical harmonics
- Correspondence
  - Via entropy
  - Registration
    - Via landmarks; thin plate splines
    - Via richer geometry

Course Order: Second Part

- Non-PDM object representations and statistics
  - Srivastava boundary normal, modulo corresp.
  - Skeletal representation (incl. [Yushkevich])
    - Medial and skeletal mathematics
    - S-reps
      - Skeletal points, spokes
      - Diffeomorphisms from ellipsoids, fitted frames
      - Taheri via swept planar cross-sections
      - Fitting to boundaries
        - » Optimization
        - » CNN
    - Cm-reps [2 lectures by P. Yushkevich]
      - Based on PDE
      - Based on splines in <u>x</u> and half-width r
      - Fitting to boundaries
  - Statistics on Riemannian manifolds [Fletcher]