Examples about conditional probability and independence

(1-1) Experiment: Throw a “2-one” dice, i.e. no 6 but two 1's.
Event A: Dice 1 is 1
Event B: Dice 1 is odd
To calculate: p{A|B} (meaning the probability of Event A occurring under the condition that Event B occurs)

p{A|B}= p{A|B} = p{x=1|x is odd} = p{x=1 and x=odd}/p{x=odd} = p{x=1}/p{x=odd} = (1/3)/(2/3) = 1/2
Note that p{A|B} != P{A}, so Event A and Event B are dependent (of course they are, by definition).

(1-2) Experiment: Throw a “2-one” dice twice, i.e. no 6 but two 1s.
Event A: Dice 1 is 1
Event B: Dice 2 is odd
To calculate: p{A|B}

We have p{x=1 and y=odd} = p{(1,1),(1,3),(1,5)} = p{(1,1)} +p{(1,3)} +p{(1,5)} = 4/36+2/36+2/36 = 2/9 (the original side for "6" also results in "1" for this special dice)
To calculate: p{A|B} = p{A|B} = p{x=1|y is odd} = p{x=1 and y=odd}/p{y=odd} = (2/9)/(2/3) = 1/3
Note that p{A} = p{x=1} =1/3, so we've verified that Event A and Event B are independent.

(2) Experiment: Rolling two independent dice. Still the “2-one” dice, i.e. no 6 but two 1's.
Event A: Dice 1 < 3
Event B: Dice 2 > 3
To calculate: p{A|B}

It's trivial that A and B are independent (two different throws have 0 impact on each other).
p{A|B}= p{x1<3|x2>3} = p{x1<3 and x2>3} / p{x2>3}
so that for a "2-one" dice:
x1={1,2}, x2={4,5}
p{x2>3} = p{x=4 or x=5} = p{x=4} + p{x=5} = 1/6+1/6 = 1/3
p{x1<3 and x2>3} = p{(1,4), (1,5), (2,4), (2,5)} = p{(1,4)}+p{(1,5)}+p{(2,4)}+p{(2,5)},
where p{(1,4)} = p(x1=1 and x2=4) = 2/36, (the original {6,4} also results in {1,4} for this special dice)
similarly p{(1,5)}=2/36;
p{(2,4)}=p{x1=2 and x2=4}=1/36
similarly p{(2,5)}=1/36;
=> p(A and B) = 1/18+1/18+1/36+1/36 = 1/6
=> p{A|B} = (1/6) / (1/3) = 1/2

p{A} = p{x=1} + p{x=2} = 1/2 = p{A|B}, so we've verified that A and B are independent.