pp. 349, Exercise Set 6.1

1. *b*. Note that $\sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$. Also 24 mod 7 = 3 because $24 - 3 = 21$ and 21 is divisible by 3. Thus

$$A = \{3, \sqrt{5^2 - 4^2}, 24 \ mod \ 7\} = \{3, 3, 3\} = \{3\}.$$

In addition, 8 mod 5 = 3 because $8 - 5 = 3$ and 3 is divisible by 3. Thus

$$B = \{8 \ mod \ 5\} = \{3\},$$

and so $A = B$. It follows that $A \subseteq B$ and $B \subseteq A$ but neither is a proper subset of the other.

d. A has three elements: a, b, and c, and B has three different elements: $\{a\}$, $\{b\}$, and $\{c\}$. So A and B have no elements in common, and thus $A \not\subseteq B$ and $B \not\subseteq A$.

f. The only integer values taken by both the sine and the cosine function are 0, 1, and -1, and they occur at $x = 0, \frac{\pi}{2}, -\frac{\pi}{2}, \pi, -\pi, \frac{3\pi}{2}, -\frac{3\pi}{2}$, etc. Because all of these numbers can be expressed as $\frac{k\pi}{2}$ for some integer k,

$$A = B = \{x \mid x = \frac{k\pi}{2} \text{ for some integer } k\}.$$

It follows that $A \subseteq B$ and $B \subseteq A$ but neither is a proper subset of the other.

3. *c*. Yes. Every element in T is in S because every integer that is divisible by 6 is also divisible by 3. To see why this is so, suppose n is any integer that is divisible by 6. Then $n = 6m$ for some integer m. Since $6m = 3(2m)$ and since $2m$ is an integer (being a product of integers), it follows that $n = 3\cdot$ (some integer), and, hence, that n is divisible by 3.

10. *f*. $B - A = \{6\}$ *g*. $B \cup C = \{2, 3, 4, 6, 8, 9\}$ *h*. $B \cap C = \{6\}$

13. *a*. True: Every positive integer is a rational number.

c. False: There are many rational numbers that are not integers. For instance, $1/2 \in \mathbf{Q}$ but $1/2 \notin \mathbf{Z}$.

e. True: No integers are both positive and negative.

f. True: Every rational number is real. So the set of all numbers that are both rational and real is the same as the set of all numbers that are rational.

g. True: Every integer is a rational number, and so the set of all numbers that are integers or rational numbers is the same as the set of all rational numbers.

h. True: Every positive integer is a real number, and so the set of all numbers that are both positive integers and real numbers is the same as the set of all positive integers.

i. False: Every integer is a rational number, and so the set of all numbers that are integers or rational numbers is the same as the set of all rational numbers. However there are many rational numbers that are not integers, and so $\mathbf{Z} \cup \mathbf{Q} = \mathbf{Q} \neq \mathbf{Z}$.

27. *b.* Yes. Every element in $\{p, q, u, v, w, x, y, z\}$ is in one of the sets of the partition and no element is in more than one set of the partition.

c. No. The number 4 is in both sets $\{5, 4\}$ and $\{1, 3, 4\}$.

e. Yes. Every element in $\{1, 2, 3, 4, 5, 6, 7, 8\}$ is in one of the sets of the partition and no element is in more than one set of the partition.

35. *c.* $B \cap C = \{2\} \quad A \times (B \cap C) = \{a, b\} \times \{2\} = \{(a, 2), (b, 2)\}$

d. $A \times B = \{a, b\} \times \{1, 2\} = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

$A \times C = \{a, b\} \times \{2, 3\} = \{(a, 2), (a, 3), (b, 2), (b, 3)\}$

$(A \times B) \cap (A \times C) = \{(a, 2), (b, 2)\}$