

pp. 161, Exercise Set 4.1

6. For example, let $a = 1$ and $b = 0$. Then

$$\sqrt{a+b} = \sqrt{1} = 1$$

and also

$$\sqrt{a} + \sqrt{b} = \sqrt{1} + \sqrt{0} = 1.$$

Hence for these values of a and b ,

$$\sqrt{a+b} = \sqrt{a} + \sqrt{b}.$$

In fact, if a is any nonzero integer and $b = 0$, then

$$\sqrt{a+b} = \sqrt{a+0} = \sqrt{a} = \sqrt{a} + 0 = \sqrt{a} + \sqrt{0} = \sqrt{a} + \sqrt{b}.$$

37. To prove the given statement is false, we prove that its negation is true. The negation of the statement is "For all integers k with $k \geq 4$, $2k^2 - 5k + 2$ is not prime."

Proof:

Suppose k is any integer with $k \geq 4$. [We must show that $2k^2 - 5k + 2$ is not prime].

We can factor $2k^2 - 5k + 2$ to obtain

$$2k^2 - 5k + 2 = (2k - 1)(k - 2).$$

But since $k \geq 4$, $k - 2 \geq 2$. Also $2k \geq 2 \cdot 4 = 8$, and thus

$$2k - 1 \geq 8 - 1 = 7.$$

This shows that each factor of $2k^2 - 5k + 2$ is a positive integer not equal to 1, and so $2k^2 - 5k + 2$ is not prime.

41. This incorrect "proof" assumes what is to be proved. The second sentence states a conclusion that follows from the assumption that $m \cdot n$ is even. The next-to-last sentence states this conclusion as if it were known to be true. But it is not known to be true. In fact, it is the main task of a genuine proof to derive this conclusion, not from the assumption that it is true but from the hypothesis of the theorem.