Quiz 3

(80') Name_____ PID_____

(8') 1. Let $A_i = [-i, i]$ what are the result for each of the following union or intersection?

(a) $\bigcup_{i=0}^{\infty} A_i =$ **(-∞,∞)** (b) $\bigcap_{i=0}^{\infty} A_i = \{\mathbf{0}\}$

(c) How many elements are there in A_1? **Infinite**

(d) How many elements are there in A_0? **1**

(8') 3. Let $B_i = \{-i, i\}$ what are the result for each of the following union or intersection?

(a) $\bigcup_{i=0}^{\infty} B_i = \mathbf{Z}$ (b) $\bigcap_{i=0}^{\infty} B_i = \emptyset$

(c) How many elements are there in B_1? **2**

(d) How many elements are there in B_0? **1**

(4') 5. Show the following equation, i.e., write some intermediate steps from the left-hand-side (LHS) to the right-hand-side (RHS) so that each step is trivial.

$$8 \cdot \left(2 \cdot 5^k - 3^k\right) - 15 \cdot \left(2 \cdot 5^{k-1} - 3^{k-1}\right) = 2 \cdot 5^{k+1} - 3^{k+1}$$

Solution:

$$\begin{aligned} & 8 \cdot \left(2 \cdot 5^k - 3^k\right) - 15 \cdot \left(2 \cdot 5^{k-1} - 3^{k-1}\right) \\ &= 16 \cdot 5^k - 8 \cdot 3^k - 30 \cdot 5^{k-1} + 15 \cdot 3^{k-1} \\ &= 80 \cdot 5^{k-1} - 24 \cdot 3^{k-1} - 30 \cdot 5^{k-1} + 15 \cdot 3^{k-1} \\ &= 50 \cdot 5^{k-1} - 9 \cdot 3^{k-1} \\ &= 2 \cdot 5^{k+1} - 3^{k+1} \end{aligned}$$

(Bonus 5') Any comments, suggestions and/or concerns about this course and/or the instructor? (E.g., until now, whether this course is harder/easier than what you expect?)