Notes 3

Quantified statements (first-order predicate logic)

    predicate symbols, predicate variables, domain, truth set

Quantifiers: universal (∀), existential (∃)

    Suppose the domain $D = \{x_1, x_2, \ldots, x_n\}$

    $\forall x \in D, P(x) \equiv P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)$

    $\exists x \in D$ such that $P(x) \equiv P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)$

Negation of predicates

    $\sim (\forall x \in D, P(x)) \equiv \exists x \in D$ such that $\sim P(x)$

    $\sim (\exists x \in D$ such that $P(x)) \equiv \forall x \in D, \sim P(x)$

Establishing truth/ falsity of quantified statements

    To show a universally quantified statement true: exhaustive enumeration

        vacuously true universally quantified statement

    To show a universally quantified statement false: a counter-example

    To show an existentially quantified statement true: an example)

    To show an existentially quantified statement false: exhaustive enumeration

Multiple Quantifiers

    Order of Quantifiers

        changing the order of different quantifiers usually changes the meaning

Arguments with quantified statements