Homework 8

Due on Monday, 6/19, 1:15 PM in class

Name________________________________ PID________________________________

Honor Code Pledge: I certify that I am aware of the Honor Code in effect in this course and observed the Honor Code in the completion of this homework.

Signature____________________________

(10') 1. In a certain state, license plates each consist of 2 capital letters followed by 3 digits.

(a) How many different license plates are there?

(b) How many different license plates are there that have no repeated letters or digits?

Solution:

(a) 26*26*10*10*10 = 676000.

(b) P(26, 2)*P(10,3) = 26*25*10*9*8 = 468000

(5') 2. In another certain state, license plates each consist of 1 to 3 capital letters followed by 1 to 4 digits (i.e., a plate may have 2 to 7 characters). How many different license plates are there that have no repeated letters or digits?

Solution:

Step1: choose the letters part, P(26,1) + P(26, 2) + P(26,3) = 26+26*25+26*25*24 = 16276.

Step2: choose the digits part,

P(10,1) + P(10,2) + P(10,3) + P(10,4) = 10+10*9+10*9*8+10*9*8*7 = 5860

So, in total, 16276*5860 = 95377360 different license plates.

(10') 3. In a certain class, three quizzes were given. Out of the 30 students in the class:

15 scored 90 or above on quiz #1,

12 scored 90 or above on quiz #2,

18 scored 90 or above on quiz #3,

7 scored 90 or above on quizzes #1 and #2,

11 scored 90 or above on quizzes #1 and #3,

8 scored 90 or above on quizzes #2 and #3,

4 scored 90 or above on quizzes #1, #2, and #3.

(a) How many scored 90 or above on at least one quiz?

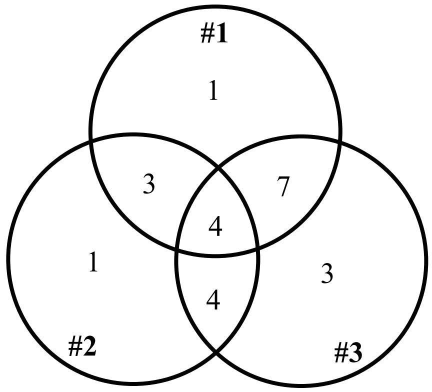
(b) How many scored 90 or above on quizzes 1 and 2 but not 3?

Solution:

Consider this problem using a diagram as attached next page.

(a) 23.

(b) 3.



(35’) 4. A club has seven members. Three are to be chosen to go as a group to a national meeting.
(a) How many distinct groups of three can be chosen?
(b) If the club contains four men and three women, how many distinct groups of three contain two men and one woman?
(c) If the club contains four men and three women, how many distinct groups of three contain at most two men?
(d) If the club contains four men and three women, how many distinct groups of three contain at least one woman?
(e) If the club contains four men and three women, what is the probability that a distinct group of three will contain at least one woman?
(f) If two members of the club refuse to travel together as part of the group (but each is willing to go if the other does not), how many distinct groups of three can be chosen?
(g) If two members of the club insists on either traveling together or not going at all, How many distinct groups of three can be chosen?

Solution:

(a) C(7,3) = (7*6*5)/(3*2*1) = 35.
(b) C(4,2)*C(3,1) = (4*3)/(2*1)*3 = 6*3 = 18.
(c) C(7,3) – C(4,3) = 35 – (4*3*2)/(3*2*1) = 35 – 4 = 31. (That is, do not contain three men.)
(d) C(7,3) – C(4,3) = 35 – (4*3*2)/(3*2*1) = 35 – 4 = 31. (That is, not all the three are men.)
(e) 31/35, approximately 88.57%.
(f) C(7,3) – C(5,1)= 35 – 5= 30.
(g) C(5,1) + C(5,3) = 5 + (5*4*3)/(3*2*1) = 5 + 10 = 15.

(20’) 5. An urn contains 3 red balls and 7 blue balls. A person selects a set of two balls from the urn at random.
(a) What is the probability that the person gets no red ball?
(b) What is the probability that the person gets exactly one red ball?
(c) What is the probability that the person gets exactly two red ball?
(d) What is the expected value of the number of red balls the person gets?

Solution:

The total number of outcomes, i.e., the size of the sample space, is C(10,2) = (10*9)/(2*1) = 45.

(a) C(7,2)/45 = (7*6)/(2*1)/45 = 21/45 = 7/15, approximately 46.67%.

(b) C(3,1)*C(7,1)/45 = 3*7/45 = 7/15, approximately 46.67%.

(c) C(3,2)/45 = (3*2)/(2*1)/45 = 3/45 = 1/15, approximately 6.67%.

(d) 0*(7/15) + 1*(7/15) + 2*(1/15) = 9/15 = 3/5 = 0.6.

(10') 6. A screening test for a certain disease is used in a large population of people of whom 1 in 1000 actually have the disease. Suppose that the false positive rate is 1% and the false negative rate is 0.5%. Thus a person who has the disease tests positive for it 99.5% of the time, and a person who does not have the disease tests negative for it 99% of the time.

(a) What is the probability that a randomly chosen person who tests positive for the disease actually has the disease?

(b) What is the probability that a randomly chosen person who tests negative for the disease actually has the disease?

Solution:

Let "Y" denote "have the disease," "N" denote "do not have the disease," "+" denote "test positive," and "–" denote "test negative."

Then, we know P(Y) = 0.001, P(N) = 0.999, P(+ | N) = 0.01, P(– | N) = 0.99, P(– | Y) = 0.005, and P(+ | Y) = 0.995.

(a)

$$P(Y\,|+) = \frac{P(Y \cap +)}{P(+)} = \frac{P(+ \cap Y)}{P(+ \cap Y) + P(+ \cap N)} = \frac{P(+\,|\,Y) \cdot P(Y)}{P(+\,|\,Y) \cdot P(Y) + P(+\,|\,N) \cdot P(N)}$$

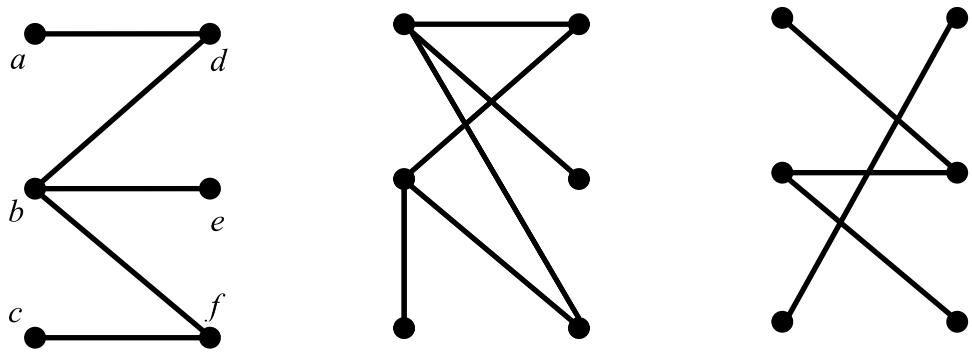
$$= \frac{0.995 * 0.001}{0.995 * 0.001 + 0.01 * 0.999} \cong 0.090578 = 9.0578\%$$

(b)

$$P(Y\,|-) = \frac{P(Y \cap -)}{P(-)} = \frac{P(- \cap Y)}{P(- \cap Y) + P(- \cap N)} = \frac{P(-\,|\,Y) \cdot P(Y)}{P(-|\,Y) \cdot P(Y) + P(-\,|\,N) \cdot P(N)}$$

$$= \frac{0.005 * 0.001}{0.005 * 0.001 + 0.99 * 0.999} \cong 0.000005 = 0.0005\%$$

(10’) 7. Consider the following graphs G_1, G_2, and G_3.



G_1 G_2 G_3

(a) What is the degree of each vertex in G_1? (The names of vertices are given.)

(b) What is the total degree of G_1?

(c) Is G_1 a tree? Is G_2 a tree? Is G_3 a tree?

Solution:

(a) *a*: 1; *b*: 3; *c*: 1; *d*: 2; *e*: 1; *f*: 2.

(b) The total degree of G_1 is 10.

(c) G_1 is a tree. G_2 is not a tree, because there is a cycle. G_3 is not a tree, because it is not connected.