Homework 7

Due on Friday, 6/16, 1:15 PM in class

Name________________________________ PID________________________________

Honor Code Pledge: I certify that I am aware of the Honor Code in effect in this course and observed the Honor Code in the completion of this homework.

Signature____________________________

(20') 1. Define a relation R from $\{a, b, c\}$ to $\{u, v\}$ as follows: $R = \{(a, u), (b, u), (c, v)\}$.

(a) Draw an **arrow diagram** for R.

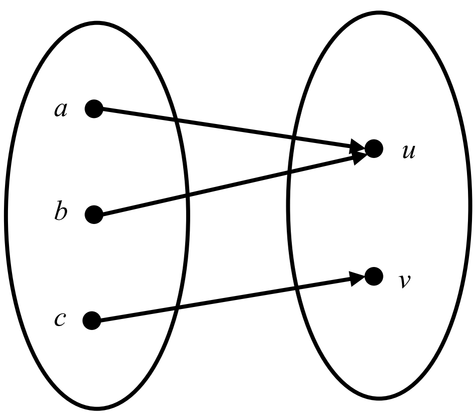
(b) Is R a function? Why or why not?

(c) Draw an **arrow diagram** for the inverse relation of R.

(d) Is the inverse relation of R a function? Why or why not?

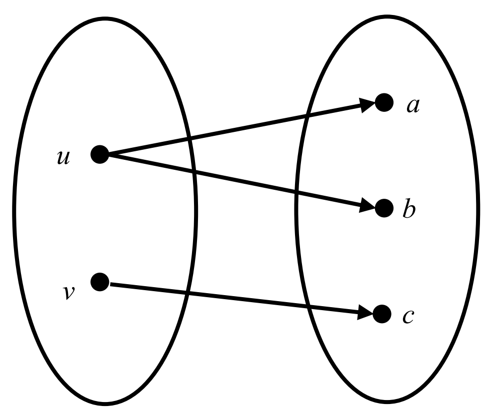
Solution:

(a)



(b) R is a function, because every element in the domain has exact one match in the co-domain.

(c)



(d) The inverse relation of R is not a function, because the element u in the domain has two matches in the co-domain.

(20') 2. Let $A = \{0, 1, 2, 3\}$ and define a relation R on A as follows:

$$R = \{(0, 2), (0, 3), (2, 0), (2, 1)\}.$$

(a) Draw the **directed graph** of R.

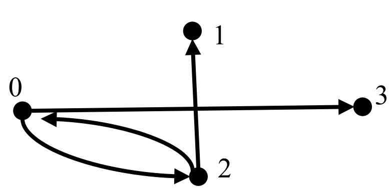
(b) Is R reflexive? Explain.

(c) Is R symmetric? Explain.

(d) Is R transitive? Explain.

Solution:

(a)



(b) R is not reflexive, because, for example, (0,0) is not included.
(c) R is not symmetric, because, for example, (0,3) is included but (3,0) is not.
(d) R is not transitive, because, for example, (0,2) and (2,1) are included but (0,1) is not.

(20') 3. Define a relation R on the set of positive integers as follows:

for all positive integers m and n, $m\ R\ n \Leftrightarrow m \mid n$.

(a) Is R reflexive? If yes, prove it; if no, disprove it by a counterexample.
(b) Is R symmetric? If yes, prove it; if no, disprove it by a counterexample.
(c) Is R transitive? If yes, prove it; if no, disprove it by a counterexample.

Solution:

(a) R is reflexive. *Proof*: for an arbitrarily chosen positive integer x, $x = 1*x$. Therefore, $m \mid m$, i.e., $x\ R\ x$. Thus, R is reflexive.
(b) R is not symmetric. Counterexample: $2\ R\ 4$ but $4 \not{R}\ 2$.
(c) R is transitive. *Proof*: suppose $x\ R\ y$ and. $y\ R\ z$, then by definition of R, $x \mid y$ and $y \mid z$. That is, $y=x*k_1$ and $z=y*k_2$ for some integers k_1,$k2$. Therefore, $z= x*k_1*k_2=(k_1*k_2)*x$, where k_1*k_2 is an integer. So, $x \mid y$, which is $x\ R\ y$. Thus, R is transitive.

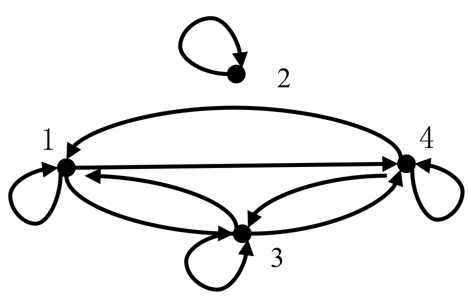
(15') 4. Let $A = \{1, 2, 3, 4\}$ and define a relation R on A as follows:

$R = \{(1, 1), (1, 3), (1, 4), (2, 2), (3, 1), (3, 3), (3, 4), (4, 1), (4, 3), (4, 4)\}$.

(a) Draw the **directed graph** of R.
(b) Is R an equivalence relation? Explain. If yes, find the distinct equivalence classes of R.

Solution:

(a)



(b) R is an equivalence relation. The two equivalence classes are [1]={1,3,4} and [2]={2}.

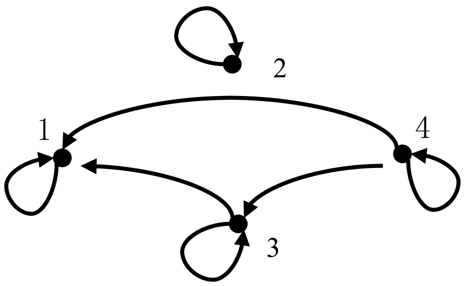
(15') 5. Let $A = \{1, 2, 3, 4\}$ and define a relation R on A as follows:

$R = \{(1, 1), (2, 2), (3, 1), (3, 3), (4, 1), (4, 3), (4, 4)\}$.

(a) Draw the **directed graph** of R.
(b) Is R a partial order relation? Explain. If yes, give a topological sorting of R.

Solution:

(a)



(b) R is a partial order relation. A topological sorting of R can be 2,4,3,1. (Other correct topological sorting includes, 4,2,3,1; 4,3,2,1; 4,3,1,2.)

(10') 6. Find the **minimum nonnegative** x, y, or z that satisfies each of the following modular arithmetic expressions.

(a) $20 \equiv x \pmod 7$ (b) $-20 \equiv y \pmod 7$ (c) $8^{10} \equiv z \pmod 7$

Solution:

(a) x=6

(b) y=1

(c) z=1 (Note that $8^{10} \equiv (8\ mod\ 7)^{10} \pmod 7$, that is $8^{10} \equiv (1)^{10} \pmod 7$.)