

# Homework 7

Due on Friday, 6/16, 1:15 PM in class

Name \_\_\_\_\_ PID \_\_\_\_\_

**Honor Code Pledge:** I certify that I am aware of the Honor Code in effect in this course and observed the Honor Code in the completion of this homework.

Signature \_\_\_\_\_

(20') 1. Define a relation  $R$  from  $\{a, b, c\}$  to  $\{u, v\}$  as follows:  $R = \{(a, u), (b, u), (c, v)\}$ .

- (a) Draw an **arrow diagram** for  $R$ .
- (b) Is  $R$  a function? Why or why not?
- (c) Draw an **arrow diagram** for the inverse relation of  $R$ .
- (d) Is the inverse relation of  $R$  a function? Why or why not?

(20') 2. Let  $A = \{0, 1, 2, 3\}$  and define a relation  $R$  on  $A$  as follows:

$$R = \{(0, 2), (0, 3), (2, 0), (2, 1)\}.$$

- (a) Draw the **directed graph** of  $R$ .
- (b) Is  $R$  reflexive? Explain.
- (c) Is  $R$  symmetric? Explain.
- (d) Is  $R$  transitive? Explain.

(20') 3. Define a relation  $R$  on the set of positive integers as follows:

$$\text{for all positive integers } m \text{ and } n, m R n \Leftrightarrow m \mid n.$$

- (a) Is  $R$  reflexive? If yes, prove it; if no, disprove it by a counterexample.
- (b) Is  $R$  symmetric? If yes, prove it; if no, disprove it by a counterexample.
- (c) Is  $R$  transitive? If yes, prove it; if no, disprove it by a counterexample.

(15') 4. Let  $A = \{1, 2, 3, 4\}$  and define a relation  $R$  on  $A$  as follows:

$$R = \{(1, 1), (1, 3), (1, 4), (2, 2), (3, 1), (3, 3), (3, 4), (4, 1), (4, 3), (4, 4)\}.$$

- (a) Draw the **directed graph** of  $R$ .
- (b) Is  $R$  an equivalence relation? Explain. If yes, find the distinct equivalence classes of  $R$ .

(15') 5. Let  $A = \{1, 2, 3, 4\}$  and define a relation  $R$  on  $A$  as follows:

$$R = \{(1, 1), (2, 2), (3, 1), (3, 3), (4, 1), (4, 3), (4, 4)\}.$$

- (a) Draw the **directed graph** of  $R$ .
- (b) Is  $R$  a partial order relation? Explain. If yes, give a topological sorting of  $R$ .

(10') 6. Find the **minimum nonnegative**  $x$ ,  $y$ , or  $z$  that satisfies each of the following modular arithmetic expressions.

- (a)  $20 \equiv x \pmod{7}$
- (b)  $-20 \equiv y \pmod{7}$
- (c)  $8^{10} \equiv z \pmod{7}$