

Homework 6

Due on Tuesday, 6/13, 1:15 PM in class

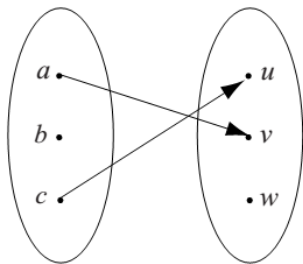
Name _____ PID _____

Honor Code Pledge: I certify that I am aware of the Honor Code in effect in this course and observed the Honor Code in the completion of this homework.

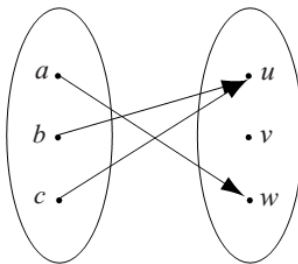
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(15') 1. Let $X = \{a, b, c\}$ and $Y = \{u, v, w\}$. Determine whether each of the following arrow diagrams defines a function from X to Y , and explain your answers in a few words.

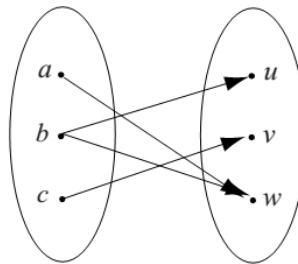
a.



b.



c.



(20') 2. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{u, v, w, x, y\}$, and define $h: X \rightarrow Y$ as follows:

$$h(1) = v, h(2) = x, h(3) = v, h(4) = v, h(5) = y.$$

(a) Draw an arrow diagram for h .

(b) Let $A = \{1, 2\}$, $C = \{x, v\}$, $D = \{w\}$, and $E = \{w, y\}$. Find

$$h(A), h(X), h^{-1}(C), h^{-1}(D), h^{-1}(E), \text{ and } h^{-1}(Y).$$

(15') 3. Let S be the set of all strings in 0's and 1's, and define a function $f: S \rightarrow \mathbf{Z}$ as follows:

for each string s in S , $f(s)$ = the number of 0's in s .

(a) What is $f(101011)$? $f(00100)$?

(b) Is f injective? Prove or give a counterexample.

(c) Is f surjective? Prove or give a counterexample.

(20') 4. Let S be the set of all strings in 0's and 1's, and define a function $g: S \rightarrow \mathbf{Z}^+ \cup \{0\}$ as follows: (Note that \mathbf{Z}^+ denotes the set of all positive integers, so $\mathbf{Z}^+ \cup \{0\}$ denotes the set of all non-negative integers.)

for all strings s in S , $g(s)$ = the number of 1's in s .

(a) What is $g(001000)$? $g(111001)$? $g(10101)$? $g(0100)$?

(b) Is g injective? Prove or give a counterexample.

(c) Is g surjective? Prove or give a counterexample.

(d) Is g a bijection? If so, find g^{-1} .

(20') 5. Define $F : \mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R} \times \mathbf{R}$ as follows: $F(x, y) = (3y - 1, 1 - x)$ for all (x, y) in $\mathbf{R} \times \mathbf{R}$.

(a) $F(0, 0) = ?$ $F(1, 4) = ?$

(b) Is F injective? Prove or give a counterexample.

(c) Is F surjective? Prove or give a counterexample.

(d) Is F a bijection? If not, explain why not. If yes, find F^{-1} .

(10') Let $f : \mathbf{R} \rightarrow \mathbf{R}$ and $g : \mathbf{R} \rightarrow \mathbf{R}$ are two functions defined as follows:

$$f(x) = x - 1; \quad g(x) = x^2 - 1.$$

Then, define $F(x) = f(g(x))$ and $G(x) = g(f(x))$.

(a) What is $F(2)$? $G(2)$?

(b) Write explicit expressions for $F(x)$ and $G(x)$. Simplify the results as much as you can.