Homework 4

Due on Monday, 6/5, 1:15 PM in class

Name_______________________________ PID_______________________________

Honor Code Pledge: I certify that I am aware of the Honor Code in effect in this course and observed the Honor Code in the completion of this homework.

Signature___________________________

(30') 1. Calculate the following sum or product. (I encourage you to include intermediate steps, which may give you partial credits in case you did the math wrong.)

(a)
$$\sum_{i=3}^{7}(i-5)$$

(b)
$$\prod_{k=0}^{4}(k!)$$

(c)
$$\left(\sum_{j=6}^{6} j\right)^2$$

(d)
$$\sum_{i=2}^{5}\sum_{j=1}^{3}(i \cdot j)$$

(e)
$$\prod_{i=1}^{3}\left(\sum_{j=1}^{i} j\right)$$

(f)
$$\sum_{i=2}^{5}\sum_{j=1}^{i}(i \cdot j)$$

(20') 2. Consider the following formula where n is an integer and $n \geq 3$:

$$\sum_{i=3}^{n} i = \frac{(n-2)(n+3)}{2}$$

(a) Expand the Left-Hand-Side of the formula. (That is, rewrite it without the "Σ" but with "…")
(b) Prove the formula by mathematical induction.

(15') 3. Prove the following statement by mathematical induction:

$7^n - 1$ is divisible by 6, for any integer $n \geq 0$.

(15') 4. Define a sequence $a_1, a_2, a_3, \ldots$ as: $a_1 = 1$, $a_2 = 3$, and $a_k = a_{k-1} + a_{k-2}$ for all integers $k \geq 3$.

Use strong mathematical induction to prove that $a_n < \left(\frac{7}{4}\right)^n$ for all integers $n \geq 1$.

(20') 5. Define a sequence $b_1, b_2, b_3, \ldots$ as: $b_1 = 2$, and $b_k = b_{k-1} + 2 \cdot 3^k$ for all integers $k \geq 2$.

(a) Calculate b_2, b_3, b_4.
(b) Use iteration to guess an explicit, closed-form formula. That is, express b_n as a function of n without "…", "Σ", or "Π".

(Hint: you might need to use the formula $1 + r + r^2 + \ldots + r^n = \frac{r^{n+1}-1}{r-1}$)

(c) Use to mathematical induction to prove the formula you derived in (b) above.