

Homework 3

Due on Friday, 6/2, 1:15 PM in class

Name _____ PID _____

Honor Code Pledge: I certify that I am aware of the Honor Code in effect in this course and observed the Honor Code in the completion of this homework.

Signature _____

Determine whether each of the following statements 1-4 is true or false. If it is true, prove it **from the definitions (nonetheless, the proof can be either direct or indirect)**; if it is false, disprove it by a counterexample.

(15') 1. \forall integers m and n , if $2m + n$ is odd then m and n are both odd.

(15') 2. For all integers n , $n^2 + n + 1$ is odd.

(15') 3. For all real numbers r , if r^3 is irrational then r is irrational.

(15') 4. For all integers a and b , if $a \mid b^2$ and $a \leq b$, then $a \mid b$.

Prove the following statement. You can use the Quotient-Remainder Theorem. That is, assume Theorem 4.4.1 on pp. 180 in the textbook is already proven.

(20') 5. For all integer n , if $3 \mid n^2$ then $3 \mid n$.

(Hint: By contradiction and by division into cases while deriving the contradiction.)

Prove the following statement. You can use statement 5 above. That is, assume you have correctly proven the statement above.

(20') 6. $\sqrt{3}$ is irrational.