Homework 2

Due on Tuesday, 5/30, 1:15 PM in class

Name_______________________________ PID_______________________________

Honor Code Pledge: I certify that I am aware of the Honor Code in effect in this course and observed the Honor Code in the completion of this homework.

Signature____________________________

(5') 1. Which of the following is a negation for "Given any real numbers *a* and *b*, if *a* and *b* are rational then *a*/*b* is rational."

(a) There exist real numbers *a* and *b* such that *a* and *b* are not rational and *a*/*b* is not rational.

(b) Given any real numbers *a* and *b*, if *a* and *b* are not rational then *a*/*b* is not rational.

(c) There exist real numbers *a* and *b* such that *a* and *b* are not rational and *a*/*b* is rational.

(d) Given any real numbers *a* and *b*, if *a* and *b* are rational then *a*/*b* is not rational.

(e) There exist real numbers *a* and *b* such that *a* and *b* are rational and *a*/*b* is not rational.

(f) Given any real numbers *a* and *b*, if *a* and *b* are not rational then *a*/*b* is rational.

Solution: (e).

(15') 2. Consider the statement "The square of any odd integer is odd."

(a) Rewrite the statement in the form "∀____*n*, ____." (Do not use the words "if" or "then")

(b) Rewrite the statement in the form "∀____*n*, if____then____."

(c) Write a negation for the statement.

Solution:

(a) ∀ odd integer *n*, n^2 is odd.

(b) ∀ integer *n*, if *n* is odd then n^2 is odd.

(c) ∃ an odd integer *n* such that n^2 is not odd.

(20') 3. Consider the statement "Everyone has a parent."

(a) Rewrite the statement using variables and both the English terms "for all" and "there exists."

(b) Rewrite the statement in the form "∀____*x*, ∃____y such that____" or "∃____*x*, ∀____y, ____."

(c) Write a negation of the statement for (a). (an English sentence)

(d) Write a negation of the statement for (b). (using the quantifiers)

Solution:

(a) For all people *x*, there exists a person *y* such that *y* is *x*'s parent.

(b) ∀ people *x*, ∃ a person *y* such that *y* is *x*'s parent.

(c) There exists a person *x* such that for all people *y*, *y* is not *x*'s parent.

(d) ∃ a person *x* such that ∀ people *y*, *y* is not *x*'s parent.

(25') 4. Consider the statement "∀ real number x, $x > 0$ only if $x^2 \geq 1$"

(a) Rewrite the statement using "if-then" instead of "only if."

(b) Write the converse, inverse, and contrapositive of the statement for (a).
(c) Write a negation of the statement for (a).

Solution:

(a) ∀ real number x, if $x^2 < 1$ then $x \leq 0$

(Or, equivalently, ∀ real number x, if $x > 0$ then $x^2 \geq 1$)

(b) *Converse*: ∀ real number x, if $x \leq 0$ then $x^2 < 1$

Inverse: ∀ real number x, if $x^2 \geq 1$ then $x > 0$

Contrapositive: ∀ real number x, if $x > 0$ then $x^2 \geq 1$

(If you write the alternative in (a):

Converse: ∀ real number x, if $x^2 \geq 1$ then $x > 0$

Inverse: ∀ real number x, if $x \leq 0$ then $x^2 < 1$

Contrapositive: ∀ real number x, if $x^2 < 1$ then $x \leq 0$)

(c) ∃ real number x such that $x^2 < 1 \wedge x > 0$

(5') 5. Is the following argument valid or invalid? Justify your answer.

All real numbers have nonnegative squares.
The number *i* has a negative square.
Therefore, the number *i* is not a real number.

Solution:

The argument is valid by Universal Modus Tollens.

(5') 6. Is the following argument valid or invalid? Justify your answer.

All prime numbers greater than 2 are odd.
The number *a* is not prime.
Therefore, the number *a* is not odd.

Solution:

The argument is invalid; it exhibits the inverse error.

(25') 7. There are three people Alice, Bob, and Chris. Each of them is either a knight, who always tells the truth, or a knave, who always lies. Two of them made the following statement.

Alice says: Bob is a knave or Chris is a knight.
Bob says: Alice is a knight if, and only if, Chris is a knave.

(a) Use a truth table to determine what each person is.
(b) Use rules of inference to justify the answer for (a).

Solution:

(a) Let the *a, b, c* denote the following statement, respectively.

a: Alice is a knight.
b: Bob is a knight
c: Chris is a knight

Furthermore, let A, B denote the statement Alice and Bob said, respectively. Then,

$A \equiv \sim b \lor c$

$B \equiv \mathrm{a} \leftrightarrow \sim c$

Therefore, we can construct the following truth table.

a	b	c	$A \equiv \sim b \lor c$	$B \equiv \mathrm{a} \leftrightarrow \sim c$	$(a \land A) \lor (\sim a \land \sim A)$	$(b \land B) \lor (\sim b \land \sim B)$
T	T	T	T	F	T	F
T	T	F	F	T	F	T
T	**F**	**T**	T	F	**T**	**T**
T	F	F	T	T	T	F
F	T	T	T	T	F	T
F	T	F	F	F	T	F
F	F	T	T	T	F	F
F	F	F	T	F	F	T

Only the third row, both facts (Alice said A and Bob said B) are true. So, Alice is a knight, Bob is a knave, and Chris is a knight.

(b) Suppose Alice is a knave. (1)

∴ What Alice said is a lie, i.e., Bob is a knight and Chris is a knave.

∴ What Bob said is the truth, i.e., Alice is a knight if, and only if, Chris is a knave.

∴ Alice is a knight (because Chris is a knave), which *contradicts* supposition (1).

∴ The negation of supposition (1) is true, i.e., **Alice is a knight**.

∴ What Alice said is the truth, i.e. Bob is a knave or Chris is a knight.

Suppose Bob is a knight. (2)

∴ What Bob said is the truth, i.e., Alice is a knight if, and only if, Chris is a knave.

∴ Chris is a knave (because Alice is a knight).

∴ Bob is a knave (because Bob is a knave or Chris is a knight), which *contradicts* supposition (2)

∴ The negation of supposition (2) is true, i.e., **Bob is a knave**.

∴ What Bob said is a lie, i.e. both Alice and Chris are knights or both Alice and Chris are knaves.

∴ **Chris is a knight** (because Alice is a knight).

Therefore, if there is any solution, it must be that Alice is a knight, Bob is a knave, and Chris is a knight. By checking the original statements, it can be found that this is indeed a solution.