

Homework 2

Due on Tuesday, 5/30, 1:15 PM in class

Name _____ PID _____

Honor Code Pledge: I certify that I am aware of the Honor Code in effect in this course and observed the Honor Code in the completion of this homework.

Signature _____

(5') 1. Which of the following is a negation for "Given any real numbers a and b , if a and b are rational then a/b is rational."

- (a) There exist real numbers a and b such that a and b are not rational and a/b is not rational.
- (b) Given any real numbers a and b , if a and b are not rational then a/b is not rational.
- (c) There exist real numbers a and b such that a and b are not rational and a/b is rational.
- (d) Given any real numbers a and b , if a and b are rational then a/b is not rational.
- (e) There exist real numbers a and b such that a and b are rational and a/b is not rational.
- (f) Given any real numbers a and b , if a and b are not rational then a/b is rational.

(15') 2. Consider the statement "The square of any odd integer is odd."

- (a) Rewrite the statement in the form " \forall ___ n , ___." (Do not use the words "if" or "then")
- (b) Rewrite the statement in the form " \forall ___ n , if ___ then ___."
- (c) Write a negation for the statement.

(20') 3. Consider the statement "Everyone has a parent."

- (a) Rewrite the statement using variables and both the English terms "for all" and "there exists."
- (b) Rewrite the statement in the form " \forall ___ x , \exists ___ y such that ___" or " \exists ___ x , \forall ___ y , ___."
- (c) Write a negation of the statement for (a). (an English sentence)
- (d) Write a negation of the statement for (b). (using the quantifiers)

(25') 4. Consider the statement " \forall real number x , $x > 0$ only if $x^2 \geq 1$ "

- (a) Rewrite the statement using "if-then" instead of "only if."
- (b) Write the converse, inverse, and contrapositive of the statement for (a).
- (c) Write a negation of the statement for (a).

(5') 5. Is the following argument valid or invalid? Justify your answer.

All real numbers have nonnegative squares.

The number i has a negative square.

Therefore, the number i is not a real number.

(5') 6. Is the following argument valid or invalid? Justify your answer.

All prime numbers greater than 2 are odd.

The number a is not prime.

Therefore, the number a is not odd.

(25') 7. There are three people Alice, Bob, and Chris. Each of them is either a knight, who always tells the truth, or a knave, who always lies. Two of them made the following statement.

Alice says: Bob is a knave or Chris is a knight.

Bob says: Alice is a knight if, and only if, Chris is a knave.

- (a) Use a truth table to determine what each person is.
- (b) Use rules of inference to justify the answer for (a).