Homework 1

Due on Thursday, 5/25, 1:15 PM in class

Name_____ PID_____

Honor Code Pledge: I certify that I am aware of the Honor Code in effect in this course and observed the Honor Code in the completion of this homework.

Signature_____

(20') 1. Write a negation for each of the following statements:

(a) The variable *S* is undeclared and the data are out of order.

(b) The variable *S* is undeclared or the data are out of order.

(c) If Al was with Bob on the first, then Al is innocent.

(d) $-5 \leq x < 2$ (where x is a particular real number)

Solution:

(a) The variable *S* is not undeclared or the data are not out of order.

(b) The variable *S* is not undeclared and the data are not out of order.

(c) Al was with Bob on the first, and Al is not innocent.

(d) $x < -5$ or $x \geq 2$

(20') 2. Write the negation, converse, inverse, and contrapositive of "If Ann is Jan's mother, then Jose is Jan's cousin."

Solution:

Negation: Ann is Jan's mother but Jose is not Jan's cousin.

Converse: If Jose is Jan's cousin, then Ann is Jan's mother.

Inverse: If Ann is not Jan's mother, then Jose is not Jan's cousin.

Contrapositive: If Jose is not Jan's cousin, then Ann is not Jan's mother.

(15') 3. Consider the following argument form:

$$\begin{array}{l} p \wedge \sim q \rightarrow r \\ p \vee q \\ q \rightarrow p \\ \therefore r \end{array}$$

Determine whether it is valid or invalid by constructing a truth table. Please include a few words explaining how the truth table supports your conclusion.

Solutions:

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The given form of argument is invalid.

					premises			*conclusion*
p	q	r	$\sim q$	$p \wedge \sim q$	$p \wedge \sim q \rightarrow r$	$p \vee q$	$q \rightarrow p$	r
T	T	T	F	F	T	T	T	T
T	T	F	F	F	T	T	T	F
T	F	T	T	T	T	T	T	T
T	F	F	T	T	F	T	T	F
F	T	T	F	F	T	T	F	T
F	T	F	F	F	T	T	F	F
F	F	T	T	F	T	F	T	T
F	F	F	T	F	T	F	T	F

Row 2 of the truth table shows that it is possible for an argument of this form to have true premises and a false conclusion.

(15') 4. Define statement variables for the following statements of truth and re-write them using the variables. Finally, determine the location of the treasure by the rules of inferences.

(a) If this house is next to a lake, then the treasure is in the kitchen.

(b) If the tree in the front yard is an elm, then the tree in the back yard is an oak.

(c) The tree in the back yard is not an oak.

(d) This house is next to a lake or the tree in the front yard is an elm.

(e) If the tree in the front yard is an elm, then the treasure is in the garage.

Solutions:

HL: this house is next to a lake

FE: the tree in the front yard is an elm

BO: the tree in the back yard is an oak

TK: the treasure is in the kitchen

TG: the treasure is in the garage

(a) HL → TK

(b) FE → BO

(c) ~BO

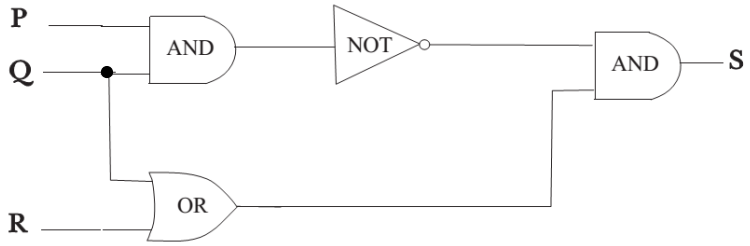
(d) HL ∨ FE

(e) FE → TG

FE → BO by (b)
~BO by (c)
∴ ~FE
HL ∨ FE by (d)
∴ HL
HL → TK by (a)
∴ TK

That is, the treasure is in the kitchen.

(10') 5. Consider the following circuit:



(a) Find the output of the circuit (S) corresponding to the input P = 1, Q = 0, and R = 1.

(b) Write the Boolean expression corresponding to the circuit (i.e., S ≡ ?).

Solutions:

(a) S = 1

(b) ~ (P∧Q) ∧ (Q∨R)

(20') 6. Construct the truth table of $(p \rightarrow q) \leftrightarrow (p \rightarrow r)$ and write its corresponding disjunctive normal form (DNF) and conjunctive normal form (CNF) expressions.

p	q	r	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \leftrightarrow (p \rightarrow r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

DNF: $(p \wedge q \wedge r) \vee (p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r)$

CNF: $(\sim p \vee \sim q \vee r) \wedge (\sim p \vee q \vee \sim r)$