Overview

• The pinhole projection model
  • Qualitative properties
  • Perspective projection matrix

• Cameras with lenses
  • Depth of focus
  • Field of view
  • Lens aberrations

• Digital cameras
  • CCD vs. CMOS
  • Color sensors
How do we see the world?

Let’s design a camera

- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?
Pinhole camera

Add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the aperture
Pinhole camera model

Pinhole model:

- Captures pencil of rays – all rays through a single point
- The point is called Center of Projection (focal point)
- The image is formed on the Image Plane
Dimensionality Reduction Machine (3D to 2D)

3D world

2D image

Point of observation

What have we lost?

• Angles
• Distances (lengths)

Slide by A. Efros
Figures © Stephen E. Palmer, 2002
Projection properties

• Many-to-one: any points along same ray map to same point in image

• Points → points
  • But projection of points on focal plane is undefined

• Lines → lines (collinearity is preserved)
  • But line through focal point projects to a point

• Planes → planes (or half-planes)
  • But plane through focal point projects to line
Projection properties

- Parallel lines converge at a vanishing point
  - Each direction in space has its own vanishing point
  - But parallels parallel to the image plane remain parallel

How do we construct the vanishing point?
One-point perspective

Masaccio, *Trinity*, Santa Maria Novella, Florence, 1425-28

First consistent use of perspective in Western art?
Perspective distortion

• What does a sphere project to?
Perspective distortion

- What does a sphere project to?
Perspective distortion

• The exterior columns appear bigger
• The distortion is not due to lens flaws
• Problem pointed out by Da Vinci
Perspective distortion

- Problem for architectural photography: converging verticals

Source: F. Durand
Perspective distortion

• Problem for architectural photography: converging verticals

  Tilting the camera upwards results in converging verticals
  Keeping the camera level, with an ordinary lens, captures only the bottom portion of the building
  Shifting the lens upwards results in a picture of the entire subject

• Solution: view camera (lens shifted w.r.t. film)

http://en.wikipedia.org/wiki/Perspective_correction_lens

Source: F. Durand
Perspective distortion

• Problem for architectural photography: converging verticals

• Result:

Source: F. Durand
Perspective distortion

• However, converging verticals work quite well for horror movies…
Modeling projection

The coordinate system

- We will use the pinhole model as an approximation
- Put the optical center \( O \) at the origin
- Put the image plane \( \Pi' \) \textit{in front} of \( O \)

Source: J. Ponce, S. Seitz
Modeling projection

Projection equations

- Compute intersection with $\Pi'$ of ray from $P = (x, y, z)$ to $O$
- Derived using similar triangles
Homogeneous coordinates

Is this a linear transformation?
- no—division by $z$ is nonlinear

Trick: add one more coordinate:

- $(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
- $(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$

Converting \textit{from} homogeneous coordinates

- $\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$
- $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$
Perspective Projection Matrix

Projection is a matrix multiplication using homogeneous coordinates:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1/f' & 0 \\
0 & 0 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} =
\begin{bmatrix}
x \\
y \\
z/ f' \\
1
\end{bmatrix}
\Rightarrow (f'/z, f'/z) \text{ divide by the third coordinate}
\]

In practice: lots of coordinate transformations…

\[
\begin{bmatrix}
2D \\
\text{point} \\
(3x1)
\end{bmatrix} =
\begin{bmatrix}
\text{Camera to} \\
\text{pixel coord.} \\
\text{trans. matrix} \\
(3x3)
\end{bmatrix} \begin{bmatrix}
\text{Perspective} \\
\text{projection matrix} \\
(3x4)
\end{bmatrix} \begin{bmatrix}
\text{World to} \\
\text{camera coord.} \\
\text{trans. matrix} \\
(4x4)
\end{bmatrix} \begin{bmatrix}
3D \\
\text{point} \\
(4x1)
\end{bmatrix}
\]
Orthographic Projection

Special case of perspective projection

- Distance from center of projection to image plane is infinite

Also called “parallel projection”

What’s the projection matrix?

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= 
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\Rightarrow (x, y)
\]
Building a real camera
Camera Obscura

- Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)
- Depth of the room (box) is the effective focal length

Source: A. Efros
Abelardo Morell

After scouting rooms and reserving one for at least a day, Morell masks the windows except for the aperture. He controls three elements: the size of the hole, with a smaller one yielding a sharper but dimmer image; the length of the exposure, usually eight hours; and the distance from the hole to the surface on which the outside image falls and which he will photograph. He used 4 x 5 and 8 x 10 view cameras and lenses ranging from 75 to 150 mm.

After he's done inside, it gets harder. "I leave the room and I am constantly checking the weather, I'm hoping the maid reads my note not to come in, I'm worrying that the sun will hit the plastic masking and it will fall down, or that I didn't trigger the lens."

From Grand Images Through a Tiny Opening, Photo District News, February 2005

http://www.abelardomorell.net/camera_obscura1.html
Home-made pinhole camera

Why so blurry?

http://www.debevec.org/Pinhole/
Shrinking the aperture

Why not make the aperture as small as possible?

- Less light gets through
- Diffraction effects…

Slide by Steve Seitz
Shrinking the aperture

2 mm

1 mm

0.6 mm

0.35 mm

0.15 mm

0.07 mm
Solution: Refraction

Snell’s law:

\[ n_1 \sin \alpha_1 = n_2 \sin \alpha_2 \]
Adding a lens

A lens focuses light onto the film
  • Rays passing through the center are not deviated
Adding a lens

A lens focuses light onto the film

- Rays passing through the center are not deviated
- All parallel rays converge to one point on a plane located at the focal length $f$
Adding a lens

A lens focuses light onto the film

- There is a specific distance at which objects are “in focus”
  - other points project to a “circle of confusion” in the image

Slide by Steve Seitz
Thin lenses

Thin lens equation: \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \)

- Any object point satisfying this equation is in focus
- What is the shape of the focus region?
- How can we change the focus region?
Depth of Field

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

Slide by A. Efros
How can we control the depth of field?

Changing the aperture size affects depth of field

- A smaller aperture increases the range in which the object is approximately in focus
- But small aperture reduces amount of light – need to increase exposure
Varying the aperture

Large aperture = small DOF
Small aperture = large DOF
Nice Depth of Field effect

Source: F. Durand
Manipulating the plane of focus

In this image, the plane of focus is almost at a right angle to the image plane.
Tilt-shift lenses

- Tilting the lens with respect to the image plane allows to choose an arbitrary plane of focus

- Standard setup: plane of focus is parallel to image plane and lens plane
Tilt-shift lenses

- Tilting the lens with respect to the image plane allows to choose an arbitrary plane of focus

- **Scheimpflug principle**: plane of focus passes through the line of intersection between the lens plane and the image plane
“Fake miniatures”

Field of View (Zoom)

From London and Upton

Slide by A. Efros
Field of View (Zoom)

From London and Upton

Slide by A. Efros
Size of field of view governed by size of the camera retina:

\[ \varphi = \tan^{-1}\left(\frac{d}{2f}\right) \]

Smaller FOV = larger Focal Length
Field of View / Focal Length

Large FOV, small f
Camera close to car

Small FOV, large f
Camera far from the car

Sources: A. Efros, F. Durand
Same effect for faces

wide-angle  standard  telephoto

Source: F. Durand
Approximating an affine camera
Real lenses
Lens Flaws: Chromatic Aberration

Lens has different refractive indices for different wavelengths: causes color fringing

Near Lens Center

Near Lens Outer Edge
Lens flaws: Spherical aberration

Spherical lenses don’t focus light perfectly
Rays farther from the optical axis focus closer
Lens flaws: Vignetting
Radial Distortion

- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens

No distortion  Pin cushion  Barrel
Digital camera

A digital camera replaces film with a sensor array

- Each cell in the array is light-sensitive diode that converts photons to electrons
- Two common types
  - Charge Coupled Device (CCD)
  - Complementary metal oxide semiconductor (CMOS)
**CCD vs. CMOS**

**CCD:** transports the charge across the chip and reads it at one corner of the array. An *analog-to-digital converter (ADC)* then turns each pixel's value into a digital value by measuring the amount of charge at each photosite and converting that measurement to binary form.

**CMOS:** uses several transistors at each pixel to amplify and move the charge using more traditional wires. The CMOS signal is digital, so it needs no ADC.


## CCD vs. CMOS

<table>
<thead>
<tr>
<th>CCD</th>
<th>CMOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mature technology</td>
<td>Recent technology</td>
</tr>
<tr>
<td>High production cost</td>
<td>Lower production cost (but…)</td>
</tr>
<tr>
<td>High power consumption</td>
<td>Low power</td>
</tr>
<tr>
<td>Higher fill rate</td>
<td>Lower fill rate (less sensitive)</td>
</tr>
<tr>
<td>Lower noise</td>
<td>Higher noise</td>
</tr>
<tr>
<td>Higher resolution</td>
<td>Lower resolution</td>
</tr>
<tr>
<td>Blooming</td>
<td>Per pixel amplification</td>
</tr>
<tr>
<td>Sequential readout</td>
<td>Random pixel access</td>
</tr>
<tr>
<td></td>
<td>Smart pixels</td>
</tr>
<tr>
<td></td>
<td>On chip integration</td>
</tr>
<tr>
<td></td>
<td>with other components</td>
</tr>
</tbody>
</table>

Source: M. Pollefeys
Color sensing in camera: Color filter array

Bayer grid

Estimate missing components from neighboring values (demosaicing)

Why more green?

Human Luminance Sensitivity Function

Source: Steve Seitz
Problem with demosaicing: color moire
The cause of color moire

Fine black and white detail in image misinterpreted as color information
Color sensing in camera: Prism

- Requires three chips and precise alignment
- More expensive
Color sensing in camera: Foveon X3

- CMOS sensor
- Takes advantage of the fact that red, blue and green light penetrate silicon to different depths

Source: M. Pollefeys

http://en.wikipedia.org/wiki/Foveon_X3_sensor

better image quality

Source: M. Pollefeys
Issues with digital cameras

Noise
  - low light is where you most notice noise
  - light sensitivity (ISO) / noise tradeoff
  - stuck pixels

Resolution: Are more megapixels better?
  - requires higher quality lens
  - noise issues

In-camera processing
  - oversharpening can produce halos

RAW vs. compressed
  - file size vs. quality tradeoff

Blooming
  - charge overflowing into neighboring pixels

Color artifacts
  - purple fringing from microlenses, artifacts from Bayer patterns
  - white balance

More info online:
  • http://electronics.howstuffworks.com/digital-camera.htm
  • http://www.dpreview.com/
Historical context

- **Pinhole model**: Mozi (470-390 BCE), Aristotle (384-322 BCE)
- **Principles of optics (including lenses)**: Alhacen (965-1039 CE)
- **Camera obscura**: Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- **First photo**: Joseph Nicephore Niepce (1822)
- **Daguerréotypes** (1839)
- **Photographic film** (Eastman, 1889)
- **Cinema** (Lumière Brothers, 1895)
- **Color Photography** (Lumière Brothers, 1908)
- **Television** (Baird, Farnsworth, Zworykin, 1920s)
- **First consumer camera with CCD**: Sony Mavica (1981)
- **First fully digital camera**: Kodak DCS100 (1990)
Next time

Light and color