ODE system Projects Analysis

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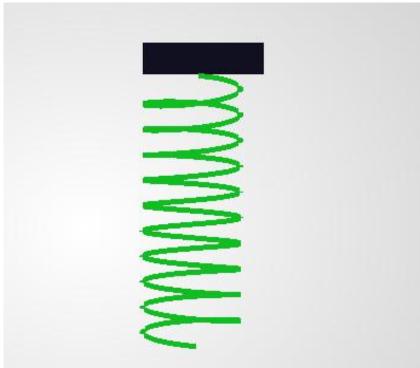


Abstract

This project aims to help us understand the basic principles of some numeric integrators like Euler and Rouge Kutta methods. They could be applied to simulate situations when the analytical solutions can't or would be very hard to obtain. Two applications projectile shooting and mass-spring are adopted to analyze the accuracy and robustness of the ODE system.

Projectile Shooting

The motion equations describe this situation are:

$$v_L = \dot{x_L},\ v_= = \dot{x_=}$$

$$\frac{dv_=}{dt} = -\frac{cv_=}{m}, \frac{dv_L}{dt} = \frac{mg - cv_L}{m}$$

P.S. I just list two dimensions, for motion with a third dimension (with azimuth), it's the same with v_L .

The following table shows the relative distance error for three methods and different time-step, here I use the trajectory with time-step=0.001s as ground-truth. (In my experiment setting, the trajectories stop when y coordinate is lower than some threshold, then I can compute the stop distance.) The last row is the distance of ground-truth.

Distance Error	RK4	RK2	Euler
0.003 s	0.0409	0.0538	0.0679
0.01 s	0.1089	0.1805	0.2522

0.05 s	**0.0362**	0.4202	0.8122
0.001 s	25.2039	25.2064	25.2089

Table 1 Parameters used in this experiment: Mass = 2.0 damping = 2.0 tilt_angle = 30.38 azimuth = 0 firepowder = 32.04

From this table, we can tell the error increase as the time-step increase, without obvious times relation. However, there is an **irregularity** (the bold number in Table 1), maybe there are some occasional situations error doesn't increase along with time-step. The error using Euler method is larger than that using RK2, and error using RK2 is larger than using RK4.

Spring-Mass system

The motion equations describe this situation are:

$$v = \dot{x}$$

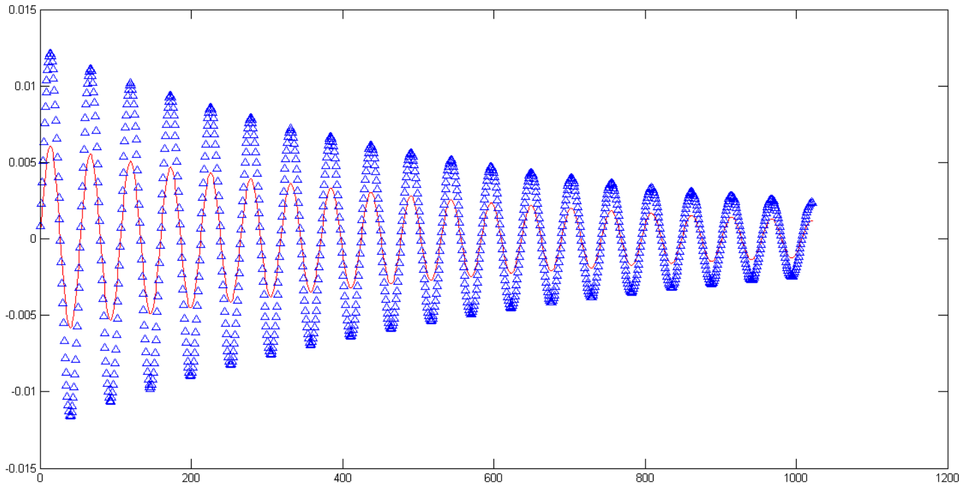
$$, \frac{dv}{dt} = \frac{mg - cv - kx}{m}$$

Table 2 shows relative error of different time-step and methods, the ground-truth use RK4 with 0.001s time-step on the time of 1.5s. From this table we can see the error increase along with time-step . It seems like RK2 and Euler have lower error than RK4.

Distance Error	RK4	RK2	Euler
0.05 s	-0.1057	-0.0689	-0.0221
0.01 s	-0.0186	-0.0122	-0.0055
0.005 s	-0.0080	-0.0049	-0.0017
0.001 s	0.2072	0. 2072	0. 2072

Table 2 Other parameters are mass=2.0 damping=0.5 hookcoef = 125.0

Then I plot the error sequence of time-step equal to 0.005s, the RK2 is more close to RK4 than



Euler method (In the above graph, red line is the error of RK2-RK4 and blue rectangle is the error of Euler-RK4).

The whole system crash when time-step is larger than 0.042s.