Exact Solid Modeling

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The Challenge

The field of solid modeling deals with the design and representation of physical objects. The two dominant representation schemata used in solid modeling are constructive solid geometry (CSG) and boundary representations (B-rep). The medial axis transform (MAT) has also been suggested as a representation for solid objects. All of these representations have different inherent strengths and weaknesses, and for most applications the flexibility to handle all of these representations is desirable. Early solid modelers were able to handle solids composed of linear boundary elements (polyhedra) and of quadric boundary elements (spheres, cylinders, etc.), and to form their Boolean combinations.

Recently, techniques developed in the field of geometric modeling have been used to model sculptured solids composed of higher-degree surfaces. Integrating such primitives into solid modelers has proven to be a challenge, requiring the representation and evaluation of intersections of parametric surfaces. Likewise, computing the medial axis transform of a polyhedral model requires computing intersections of quadric surfaces. Developing robust and accurate algorithms and implementations for these problems has been a major challenge.

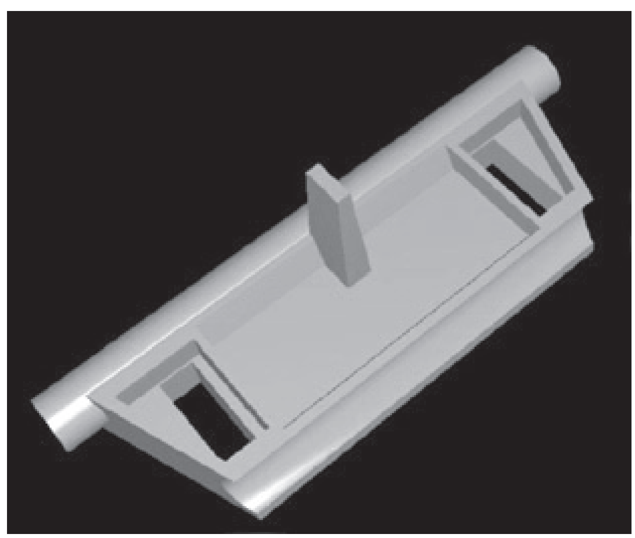
Highlights

- **Nonlinear geometric computations**
- **Accurate algorithms and implementations**
- **Robust computations**
- **Boundary evaluation**
- **Medial axis transform**

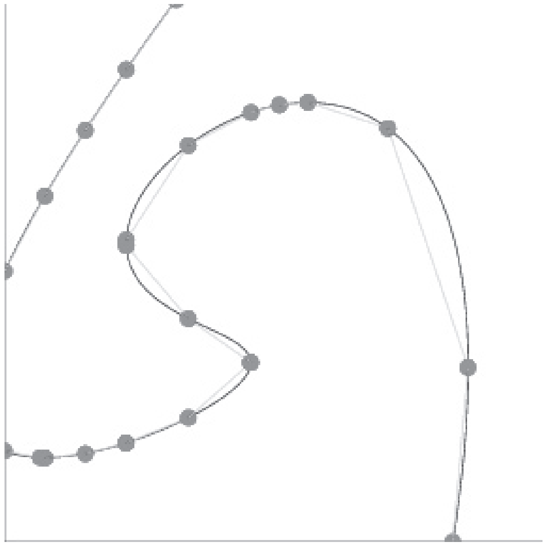
The Approach

We have been working on accurate algorithms and techniques to handle degenerate configurations for boundary evaluation and medial axis computation. We have developed exact representations for representing the solid models. Our ESOLID solid modeling system computes the B-rep of a sculptured CSG model. Our EMAT system computes the medial axis transform of a polyhedral model. ESOLID and EMAT share a common subsystem: MAPC, a publicly-available C++ library, for manipulating algebraic points and curves. The systems attempt to address accurate and robust computation. In geometric computing, a system based on floating-point arithmetic is susceptible to two common problems. First, round-off error in geometric objects can lead to inconsistent output or crashes. Secondly, degenerate and near-degenerate configurations (such as four surfaces meeting at a point) can be difficult to detect and complicated to handle. We avoid round-off error by using exact computation and representations. Exact arithmetic also aids in the detection and treatment of degenerate configurations. It is widely perceived that exact arithmetic is very slow. Our major focus has been on developing specialized representations and algorithms to improve performance. This involves isolating a few key kernel routines which govern the efficiency of the overall program. We use algebraic techniques and combinations of exact and floating point arithmetic to speed up our kernel functions,

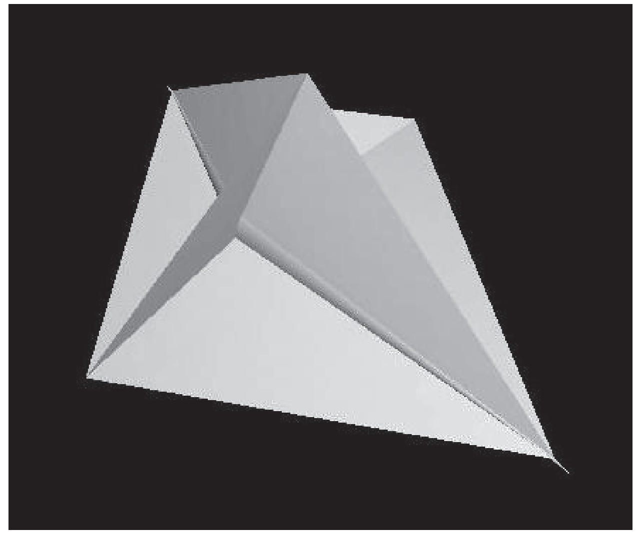
A CSG model of a Bradley fighting vehicle. (Model courtesy of the U.S. Army Research Lab).



A link in the left track, in boundary representation.



A curve parsed into monotonic segments by MAPC.



The medial axis of a simple polyhedron.

yielding in some cases performance that is within an order of magnitude of purely floating-point implementations.

- www.cs.unc.edu/~geom/MAPC
- www.cs.unc.edu/~geom/ESOLID
- www.cs.unc.edu/~geom/EMAT

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Selected Publications

Keyser, J., S. Krishnan, and D. Manocha. "Efficient and Accurate B-rep Generation of Low Degree Sculptured Solids Using Exact Arithmetic: I–Representations," *Computer-Aided Geometric Design*, 16(9), October 1999, 841–859.

Keyser, J., S. Krishnan, and D. Manocha. "Efficient and Accurate B-rep Generation of Low Degree Sculptured Solids Using Exact Arithmetic: II–Computation," *Computer-Aided Geometric Design*, 16(9), October 1999, 861–882.

Hoff, K., T. Culver, J. Keyser, M. Lin, and D. Manocha. "Fast Computation of Generalized Voronoi Diagrams Using Graphics Hardware," *Computer Graphics: Proc. SIGGRAPH '99*, Los Angeles, Calif., 8–13 August 1999, 277–285.

Keyser, J., T. Culver, D. Manocha, and S. Krishnan. "MAPC: A Library for Efficient and Exact Manipulation of Algebraic Points and Curves," *Proc. 15th Annual Symposium on Computational Geometry*, 1999.

Culver, T., J. Keyser, and D. Manocha. "Accurate Computation of the Medial Axis of a Polyhedron," *Proc. Fifth Symposium on Solid Modeling and Applications*, 1999.

Krishnan, S., M. Gopi, D. Manocha, and M. Mine. "Interactive Boundary Computation of Boolean Combinations of Sculptured Solids," *Proc. Eurographics '97*.

Keyser, J., S. Krishnan, and D. Manocha. "Efficient and Accurate B-rep Generation of Low Degree Sculptured Solids Using Exact Arithmetic," *Proc. ACM Solid Modeling '97*.

Krishnan, S., and D. Manocha. "An Efficient Surface Intersection Algorithm Based on Lower Dimensional Formulation," *ACM Transactions on Graphics*, 16(1), 1997, 74–106.

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